IMPROVED USER EQUILIBRIUM BASED METHOD FOR ESTIMATING TRIP TABLES

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ABSTRACT: An optimal trip table correction function for the user equilibrium based trip table estimation problem is derived and used in place of Turnquist and Gur’s heuristic trip table correction function. The derivation of this new correction function is based on a new concept—path equivalent capacity—that unifies the individual capacities of the links in a path into a single path capacity. Once a path between two zones is represented as a link with a single capacity, the travel time of the path can be computed directly for any path flow. This permits an optimal trip table correction function to be determined at each iteration of Turnquist and Gur’s trip table algorithm. Test results show that the new method yields more accurate and consistent solution trip tables than the heuristic method.

INTRODUCTION

Link flows are often available from a number of telemetric sources. If accurate estimates of trip tables can be obtained from observed link flows, the cost of origin-destination (O-D) data needed for transportation planning and modeling exercises might be greatly reduced relative to survey methods. Furthermore, the capacity to provide rapid updates of trip tables is relevant to a number of advanced traveler management information system applications.

Nguyen (1977) suggested the first mathematical program for estimation of a trip table from user equilibrium link volumes. The solution to Nguyen’s program is a trip table that, when assigned to the network, replicates the observed link volumes. Nguyen’s work is distinctive because it takes into account link capacity or congestion and eliminates the assumption of proportional assignment. But, subsequent work by Turnquist and Gur (1979) and LeBlanc and Farhangian (1982) indicated that Nguyen’s solution algorithm is inefficient when applied to large networks because it updates a single trip interchange at each iteration.

Turnquist and Gur (1979) improved Nguyen’s approach by developing an iterative descent algorithm, with more efficient search procedure. Under their approach, a trip table correction procedure defines the direction of the next feasible solution at each iteration by correcting the current trip table. Turnquist and Gur reported that this trip table correction procedure is a crucial element of the algorithm, determining the efficiency of the technique and the nature of the final solution. They also pointed out that the trip table correction procedure must be regarded as heuristic because there is no proof that their procedure will always converge to a solution.

Gur (1983), Han and Sullivan (1983), Fisk (1988, 1989), Oh (1992), and Yang (1995) carried out further elaborations and modifications of Nguyen’s and Turnquist and Gur’s models. These extensions focus on improving the algorithm’s efficiency, or on coping with the problem of underspecification in the original program. Little progress has been made in improving the trip table correction procedure in Turnquist and Gur’s algorithm, despite the central role of this element.

In this paper, it is suggested that a new solution search method be used. The most important feature of the new method is that it computes the optimal search direction at each iteration. This modification replaces Turnquist and Gur’s heuristic trip table correction procedure.

This paper consists of five parts. The first briefly reviews Turnquist and Gur’s algorithm. The second section derives the optimal solution search direction. The third section describes the generation of a test data set. The fourth section provides numerical performance comparisons for the heuristic and the convergent methods. Conclusions are listed in the final section.

NEW TRIP TABLE CORRECTION FUNCTION

Background

Turnquist and Gur’s (1979) mathematical program for finding an unknown O-D matrix that replicates an observed set of network link volumes can be stated as

\[ \text{min } \left[ \sum \left( \int t_a(x) \, dx \right) - \sum \bar{u}_j T_j \right] \]  

subject to

\[ T_j - \sum \bar{h}_j^i = 0 \quad \text{for each O-D pair } j \]  

\[ \sum \bar{d}_a^k h_j^k = f_a \quad \text{for each link } a \]  

where \( f_a = \text{observed flows on link } a \); \( t_a(x) = \text{travel time function for link } a \); \( \bar{u}_j = \text{observed O-D travel time for trip interchange } j \); \( T_j = \text{trips for interchange } j \); \( h_j^i = \text{number of trips of interchange } j \) using path \( k \); and \( d_a^k = 1 \), if link \( a \) is in path \( k \) for interchange \( j \) and 0, otherwise. Eq. (2) imposes trip interchange conservation for each O-D pair. Eq. (3) requires that the flow on each link consists of the sum of the flows on all paths that use the link.

Turnquist and Gur’s solution algorithm includes the following steps:

- **Step 1.** Let \( i \) be an iteration counter = 1.
- **Step 2.** Specify an initial trip table \( T^{i-1} \) and a volume-delay function for each link.
- **Step 3.** Based on the observed link travel time values, find \( \bar{u}_j \), the observed O-D travel time for trip interchange \( j \).
Step 4. Assign $T^{i+1}$ to the unloaded network by using the free flow travel time value $u_0^f$ to obtain a set of link volumes $f^{i+1}$.

Step 5. Determine link travel time $u_j^f$ at the current volume $f^i$, and rebuild minimum travel time trees for all O-D pairs.

Step 6. Given $T_j$, $u_j$, $u_j^f$ and $u_0^f$, find a correction (improved) trip table based on the differences between the current path travel times and the observed travel times.

Compute elements:

$$V_j = T_j \times \left( 1 + 2 \times \frac{\bar{u}_j - u_j}{u_j - u_0^f} \right) \quad \text{if} \ \bar{u}_j > u_j \quad (5a)$$

$$V_j = 0 \quad \text{if} \ \bar{u}_j \leq u_j \quad (5b)$$

Step 7. Assign $V_j$ to the minimum travel time tree built in Step 5. This provides a new set of link volumes $s_l$.

Step 8. Find a weight $0 \leq r' \leq 1$ such that new link flows and trip interchange requirements

$$[(f^{i+1}, T^{i+1}) = r'(s', V') + (1 - r') \ast (f^i, T^i)]$$

minimizes the objective function [(1)].

Step 9. If the preset convergence criterion is met, stop. Otherwise, $i = i + 1$, and return to Step 5.

The procedure produces a feasible trip interchange matrix at every iteration. Steps 1 and 2 are initialization. Steps 3–6 find the direction of search for the next feasible solution. At Steps 7 and 8, the moving step size is determined, and the solution is moved to the next feasible solution. The final step tests for convergence. This algorithm is a typical Frank-Wolfe solution procedure, except for Step 6. Step 6 defines the direction of the search for the next feasible solution. Turnquist and Gur (1979) and Gur (1983) showed that the trip correction function strongly affects efficiency of the algorithm and the nature of the solution. They emphasized that the trip table update procedure [(5a) and (5b)] was selected heuristically based on a number of empirical tests.

**Derivation of Path Equivalent Capacity**

Path equivalent capacity $\kappa$ is a new concept used to derive the new trip table correction equation for Turnquist and Gur’s algorithm. In the context of U.S. Bureau of Public Roads (BPR) style travel time function, the path equivalent capacity aggregates the separate capacities of the links in a path into a single representative path capacity. Each such path has a unique path equivalent capacity regardless of the number of links in the path.

Any measure of path equivalent capacity is required to satisfy the following two conditions:

- **Condition 1**: There should be one and only one value of $\kappa$ that represents the capacity of a path with multiple links.
- **Condition 2**: The travel time of a path computed relative to $\kappa$ should be the same as the sum of the travel times across each link in the path, regardless of the number of links in the path and the level of traffic demand.

Assume that flow $f$ is assigned to an empty path. Relying on the BPR function, express the path travel time in terms of path equivalent capacity $\kappa$ as

$$\tau = u_0^f \left[ 1 + \alpha \left( \frac{f^p}{\kappa} \right)^{\beta} \right]$$

where $\tau = \text{travel time along the path expressed in terms of} \ \kappa$ instead of link capacities; $u_0^f = \text{free flow travel time along the path}; f = \text{flow on the path}; \ \kappa = \text{path equivalent capacity}; \ \alpha$ and $\beta = \text{parameters for the BPR link performance function}$.

The objective is to relate to the capacities of the individual links $l$ that comprise the path to the new parameter describing path equivalent capacity $\kappa$. Assume that a path is composed of $N$ links. The path travel time is also the sum of the travel times of each link in the path

$$t = \sum_{i=1}^{N} t_i^l = \sum_{i=1}^{N} t_i^l \left[ 1 + \alpha \left( \frac{f^l}{c_i} \right)^{\beta} \right]$$

(8)

where $t = \text{path travel time}; t_i = \text{travel time for link} \ l \ (l = 1, 2, 3, \ldots, N); t_i^l = \text{free flow travel time for link} \ l; f = \text{uniform flow on each link in the path}; c_i = \text{capacity of link} \ l; \ \text{and} \ N = \text{number of links in the path}$. To satisfy Condition 2, the $\tau$ value in (7) should be identical to the $t$ value in (8). It follows that

$$u_0^f \left[ 1 + \alpha \left( \frac{f^p}{\kappa} \right)^{\beta} \right] = \sum_{i=1}^{N} t_i^l \left[ 1 + \alpha \left( \frac{f^l}{c_i} \right)^{\beta} \right]$$

(9)

Dividing both sides of the product $u_0^f f^p$ and solving for $\kappa$ gives

$$\kappa = \left( \frac{u_0^f}{\sum_{i=1}^{N} t_i^l / (c_i)^{\beta}} \right)^{1/\beta}$$

(12)

The nonnegative root of (12) expresses the path equivalent capacity in terms of the capacities and free flow travel times on the links in the path. This satisfies Conditions 1 and 2.

**Derivation of New Trip Table Correction Function**

Let $u_i^j$ denote the shortest path travel time for trip interchange $j$ at the current iteration. Let $T^*_{ij}$ denote the number of trips that, when assigned to the same shortest path, reproduces the current path travel time $u_i^j$. Then, $u_i^j$ can be expressed in terms of $T^*_{ij}$ and the shortest path equivalent capacity $\kappa_j$ as follows:

$$u_i^j = u_0^f \left( 1 + \alpha \left( \frac{T^*_{ij}}{\kappa_j} \right)^{\beta} \right)$$

(13)

where $u_0^f = \text{path free flow travel time}$. Given a nonnegative real number $\lambda$, satisfying

$$T^*_{ij} = \lambda \cdot T_j$$

(14)

and substituting (14) into (13) gives

$$u_i^j = u_0^f \left( 1 + \alpha \left( \frac{\lambda T_j}{\kappa_j} \right)^{\beta} \right)$$

(15)

where $T_j$ = number of trips for $j$ at the current iteration; and $\lambda = \text{nonnegative multiplier}$. Eq. (15) represents the current travel time on the shortest path as a function of a path equivalent capacity $\kappa_j$ and the number of trips for trip interchange $j$ denoted as $T_j$. Because the current travel time on the shortest path is, in general, not the same as the target (observed) travel
time, it can be adjusted in the optimal direction by changing the trip interchange value (i.e., toward the target travel time). When the adjusted trip interchange value \( V'_j \) replaces \( T'_j \) in (15), the result should be the observed travel time on the shortest path for trip interchange \( j \). That is, the following equation must hold for the trip interchange value:

\[
\bar{u}_j = u'_j \left( 1 + \alpha \frac{\lambda V'_j}{\kappa_j} \right)^{\beta_j}
\]  

(16)

where \( \bar{u}_j \) = observed shortest path travel time for \( j \); and \( V'_j \) = corrected number of trips for \( j \) at the current iteration. Eqs. (15) and (16) can be rewritten, respectively, as follows:

\[
T'_j = \frac{\kappa_j}{\lambda} \times \left( \frac{1}{\alpha} \left( \frac{u'_j - u_j}{u_j^e - u_j} \right)^{1/\beta_j} \right)
\]  

(17)

\[
V'_j = \frac{\kappa_j}{\lambda} \times \left( \frac{1}{\alpha} \left( \frac{\bar{u}_j - u'_j}{u'_j - u_j} \right)^{1/\beta_j} \right)
\]  

(18)

Because the travel time of a used path for trip interchange \( j \) is always greater than its free flow travel time, \((u'_j - u_j^e)\) is always greater than zero. Thus, (18) can be divided by (17), giving

\[
\frac{V'_j}{T'_j} = \left( \frac{\bar{u}_j - u'_j}{u'_j - u_j} \right)^{1/\beta_j}
\]  

(19)

or

\[
V'_j = T'_j \times \left( \frac{\bar{u}_j - u'_j}{u'_j - u_j} \right)^{1/\beta_j}
\]  

(20)

Eq. (20) replaces Turnquist and Gur’s heuristic trip table correction function.

**Comparison of New Equation to Turnquist and Gur’s Correction Equation**

The new trip table correction equation has three important features relative to Turnquist and Gur’s trip correction functions. First, the adjustment direction of the trip table correction defined by (20) is optimal because the number of trips \( V'_j \), when assigned to the current shortest path for interchange \( j \) in place of \( T'_j \), replicates the observed path travel time of \( j \). This condition is enforced by (16).

Second, the new trip table correction function is a continuous multiplicative form, whereas Turnquist and Gur’s equations are additive and discontinuous, depending on the value of the path travel time at the current iteration. The new procedure never forces entries in the updated trip table below zero because the observed travel time for the trip interchange is always greater than the free flow travel time. In (20), \( V'_j \) is greater than \( T'_j \) when the current path travel time is smaller than the observed travel time (i.e., \( u'_j < \bar{u}_j \)). In addition, \( V'_j \) is smaller than \( T'_j \) when the current path travel time is greater than the observed travel time (i.e., \( u'_j > \bar{u}_j \)). When the current path travel time is the same as the observed travel time (i.e., \( u'_j = \bar{u}_j \)), \( V'_j \) remains equal to \( T'_j \). Under Turnquist and Gur’s approach, (5b) is needed to prevent trip table corrections from becoming negative when the current path travel time \( u'_j \) exceeds the exogenously observed path travel time \( \bar{u}_j \).

Fig. 1 shows the differences between the two trip correction functions. In the figure, the new function is a single continuous curve depicted by A’-B’-C’. Turnquist and Gur’s function is a combination of curve A-B and the horizontal line C-D. The vertical distances between the two curves measure the differences in the trip corrections associated with the two models. Turnquist and Gur’s model resulted in a large error when the path travel time at the current stage \( u'_j \) is slightly greater than

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**TEST DATA**

This new trip table correction method is compared to Turnquist and Gur’s original procedure in the case of a simple synthetic network and representative trip tables. The true trip table is assigned to the network under user equilibrium conditions. The assigned link volumes are used as the observed volume data.

The availability of the true trip table permits a comparison with the estimated trip tables. This provides the advantage of evaluating results by referring to trip table errors instead of link volume errors as measures of model performance. In general, there are a number of different trip tables that, when assigned to the network, can replicate the observed link volumes. Therefore, replicating link volumes provides incomplete information about the quality of the estimated trip table.

**Network Data**

The synthetic network created to test the algorithm has five zones, 16 nodes, and 64 links (Fig. 2). The availability of multiple paths for each trip interchange mimics the conditions in a complex urban network. The free flow travel times and capacities for each link are defined so that 32 links function as part of major arterials, and the other 32 links function as minor arterials or collectors. The capacities of major arterials range between 2,500 and 3,500 vehicles/h, and those of the minor arterials or collectors range from 750 to 1,300 vehicles/h. The travel time on each link is given by the BPR performance function, with typical parameter values \( \alpha = 0.15 \) and \( \beta = 4.0 \) (Sheffi 1985). The true trip table is given in Table 1.

**Link Volume Data**

In most real cases, the observed link volumes are counted directly at each link. The most common source for volume
error-free user equilibrium flows computed based on a known (true) trip table.

**Initial Trip Tables (ITTs)**

Three different ITTs are used as the model input data to assess the performance of the new algorithm and sensitivity to starting conditions. ITT-1 is the same as the true trip table. It is used to test how accurately both models can replicate the true trip table. This measures the intrinsic limitation of the models. The other ITTs are obtained by increasingly large perturbations of the entries in ITT-1. These other ITTs are used as common starting conditions. The total number of trips in ITT-2 accounted for about 20% less than the total number of trips in the true trip table. ITT-2 is obtained by decreasing each of the cell values of the true trip matrix by 20% and then randomly increasing or decreasing each value by up to 10% based on the computer generated random numbers. ITT-2 represents an outdated trip table for an area in which travel demand has increased (Table 2).

ITT-3 is an ITT in which the number of trips are increased relative to the true trip table by 20%, and then each cell value in the matrix is increased or decreased randomly up to 10% (Table 3).

**ANALYSIS OF TEST RESULTS**

The three trip tables are used as input data for both versions of the model. The same network data and equilibrium link volume data are used in each case. A maximum of 100 iterations is executed in each case.

**Measures of Model Performance**

Two measures are used to quantify the performance of each procedure. The first performance measure is trip root-mean-squared error (trip RMSE). This is the average number of squared trip errors in the nonzero cells of the trip table matrix

\[
\text{trip RMSE} = \sqrt{\frac{\sum_{j} (T_{ij} - \bar{T}_{ij})^2}{n}}
\]  

(21)

where $T_{ij}$ = number of trips estimated for nonzero trip interchange $j$; $\bar{T}_{ij}$ = true number of trips for nonzero trip interchange $j$; and $n$ = number of nonzero trip interchanges.

The second measure of performance, link volume root-mean-squared-error (volume RMSE), is computed from the difference between estimated link volumes and the equilibrium link volumes associated with the true trip matrix

\[
\text{volume RMSE} = \sqrt{\frac{\sum_{l} (V_{il} - \bar{V}_{il})^2}{L}}
\]  

(22)

where $V_{il}$ = volume estimated by the model for link $l$; $\bar{V}_{il}$ = user equilibrium volume on link $l$; and $L$ = number of links.

**TABLE 4. Trip RMSEs**

<table>
<thead>
<tr>
<th>Method</th>
<th>ITT-1 (1)</th>
<th>ITT-2 (2)</th>
<th>ITT-3 (3)</th>
<th>ITT-3 (4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Turque and Gur (1)</td>
<td>525.8</td>
<td>429.6</td>
<td>629.9</td>
<td></td>
</tr>
<tr>
<td>New (2)</td>
<td>125.3</td>
<td>175.2</td>
<td>155.0</td>
<td></td>
</tr>
<tr>
<td>Difference* (%)</td>
<td>-76.2</td>
<td>-59.2</td>
<td>-75.4</td>
<td></td>
</tr>
</tbody>
</table>

*[(2) - (1)]/[(1) × 100].
Comparing Trip Errors

The trip RMSE values for both methods across the three different ITTs appear in Table 4. For ITT-1, the ITT is congruent with the true trip table; thus one can expect small trip RMSE values. The trip RMSEs for Turnquist and Gur’s algorithm and for the new algorithm are 52.8 and 125.3, respectively. Thus, with perfect initial information, the new algorithm reduced trip RMSE by 76.2%. The trip RMSE values from the new algorithm gave RMSE reductions of 59.2 and 75.4% for the imperfect starting conditions defined by ITT-2 and ITT-3. Fig. 3 compares the results for the two methods. The new method estimated consistently better trip tables than Turnquist and Gur’s original procedure and appears less sensitive to ITTs. Surprisingly, when the true trip table is used as an initial condition, Turnquist and Gur’s procedure results in an estimate with a larger trip RMSE than is observed when a perturbed trip table is used as the initial condition. This is contrary to many previous observations (Turnquist and Gur 1979; Gur 1983; Han and Sullivan 1983) that user equilibrium based trip table estimation algorithms are most sensitive to the ITT and have a tendency to converge to a solution that is nearest to the ITT. These results indicate that this is not necessarily the case.

Comparing Link Volume Errors

Link volume error is frequently used to measure the performance of mathematical trip table estimation models. Comparing the link volume RMSE with the corresponding trip RMSE helps assess the appropriateness of the volume RMSE as a measure of model performance. The link volume RMSEs for both models are shown in Table 5.

The link volume RMSEs associated with the new procedure are all greater than the corresponding errors in Turnquist and Gur’s outputs, although all of these errors are small enough to accept. These results contrast with the trip RMSE comparisons in Table 2. This suggests that trip RMSEs and the link volume RMSEs provide inconsistent indications of model performance. The new method resulted in more accurate trip table estimation than the previous model, but at the same level of link volume RMSE. A smaller link volume error does not necessarily mean that the model producing this error is performing better. The link volume error measure alone may well be inadequate for evaluation of the trip estimation model.

CONCLUSIONS

It has been suggested here that a new trip table adjustment equation be used for the user equilibrium based trip table estimation algorithm. Compared to this new method, Turnquist and Gur’s method tends to overestimate the trip table adjustments needed at each iteration.

The new method resulted in better solutions in terms of trip RMSE when tested against a true, known trip table and two scenario initial trip tables representing progressively perturbed (outdated) data. Test results also show that the new method yielded more consistent outputs (estimated trip tables), demonstrating considerably less sensitivity to the ITT table data. The new method performs less well in terms of link volume RMSE, suggesting that the link volume error is a weaker measure of model performance than previously assumed.

APPENDIX. REFERENCES


