MILLS' URBAN SYSTEM MODELS: PERSPECTIVE AND TEMPLATE FOR LUTE (LAND USE / TRANSPORT / ENVIRONMENT) APPLICATIONS *

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ABSTRACT. Linear programming models of spatial economy (Mills, 1972, 1974a, 1974b, 1975) have prompted the development of several diverse extensions. The purpose of this paper is to review and assess the impact of Mills' model on the development of its derivatives, and to evaluate the role of these models in the development of LUTE (Land Use / Transport / Environment) formulations. These models are assessed and reviewed by dividing them into four categories: (1) static urban linear programming models; (2) static nonlinear programming models; (3) static regional scale models; and (4) dynamic models, including intertemporal and sequential programming models. An extended Mills model is proposed that combines intertemporal production and transportation with pollution control—a prototype LUTE application.

MOTIVATION

Scientific efforts to model land use and transportation systems, either separately or together, have met with mixed success. Theoretical perspectives have improved and continue to become more sophisticated, but the cost of gathering and processing the data necessary to operationalize and apply these ideas remains a restrictive barrier. Large scale modeling efforts persist despite limited data and outright suspicion on the part of decision makers and other potential clients (Wegener, 1994), but it is clear that scholars continue to be frustrated by their attempts to relate theory to policy (Bailly & Coffey, 1994). Land use / transportation models do not yet provide the desired bridge. Integrating environmental models with land use / transportation models will not simplify matters.

Why then contemplate extending the objectives of a perspective already taxed to the limit of its means? In simplest terms, the need remains, and may be growing. The formulation of land use, transportation, and environmental policies will continue regardless of whether the effects of investments in the built environment can be adequately modeled or not. The same cannot be said of these investments. Environmental and quality of life concerns, coupled with limitations on private property rights, may well preclude socially useful investments in

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infrastructure and buildings unless the effects of these investments can be predicted in acceptable ways. Consequently, regional scientists, urban planners, transportation specialists, and other experts strive to provide decision makers with tools able to predict alternative states for urban systems in sufficient detail to permit comparison, evaluation, and choice.

This objective is so daunting that, with important exceptions cited by Wegener (1994) the response of researchers and academics seems to be to shift away from a unified (or even integrated) perspective. This is understandable. As the objective of reliable prediction is taken more seriously, there is a natural tendency to define tighter system boundaries. Recent developments in the study of transient, i.e. dynamic network equilibrium flows provide an excellent example. Developments in this difficult area tend to ignore potential feedbacks between the land use and transportation systems, and focus on a (hopefully) more relevant representation of network resources.

If such developments are useful, is it germane to consider moving in the opposite direction by deliberately broadening our modeling perspective and attempting to combine treatments of land use, transport and the environment? After all, this shift may involve the study of systems too large to measure at the level of detail needed to produce an empirically based prediction. Is the investment worth the intellectual capital required to proceed?

We think so, though sometimes for different reasons. We are both influenced by the work of Edwin Mills. This influence is exerted at a theoretical level, and in terms of his success in mining operations research techniques for insight into urban economic systems. We remain interested in the approaches Mills defined and we developed in our dissertations and subsequent work. This interest reflects our selfishness about our own states of information. Mills’ (Mills, 1972) original linear programming model of urban land use and transport provides a very complete, accessible conceptualization of urban systems. From a computational perspective, Mills’ examples are outdated. From a theoretical point of view, the work remains a refreshing blend of urban economics, operations research, duality, and policy. The model is virtually a paradigm unto itself, integrating easily with the work of other scholars in a way that organizes and leverages a wide range of different contributors.

Faced with the question of addressing how land use / transportation models might be extended to include new environmental elements, we have returned to the foundations of our training and reviewed Mills’ work and some precursor formulations relevant to his work and our own. This is a very basic way to proceed, but the question is important and we are risk averse (especially Moore). We conclude our review with an extension of Mills’ approach that, combined with the work of others, provides a tractable beginning for modeling LUTE problems.

**OPERATIONS RESEARCH AND URBAN PLANNING**

For four decades linear programming has held a pre-eminent position in economics and operations research. This is primarily due to the utility of the simplex method, a solution algorithm developed by George Dantzig (1963). Many sophisticated applications of linear programming have been developed, including numerous resource allocation problems in urban planning. Much of the impetus for these developments was provided by the pathbreaking text of Dorfman et al. (1958) on linear programming and economic efficiency. This work is especially important in the context of land use and transportation planning, because these efforts involve organizing and managing the market for urban land. The connection between linear programming and general equilibrium economics provides urban planners with a new view of the field’s objectives.
The influence of operations research (OR) on spatial resource allocation problems can be traced to a small set of seminal papers. The nonlinear program of Beckmann et al. (1956) identifies competitive equilibrium flows for a transportation network with congestive links. Koopmans and Beckmann (1957) formulated a quadratic assignment problem (QAP) that minimizes the location and interaction costs for a set of discrete activities located on a network. The linear program (LP) of Herbert and Stevens (1960) explores equilibrium conditions in the housing market. Hakimi (1964) considers the problem of locating one or more discrete facilities on a network to minimize either the sum of distances or the maximum distance between facilities and network nodes. And finally, Mills (1972) captures simultaneous equilibria in the markets for land and transportation in a single, highly tractable linear program.

Much important work has followed these efforts, and operations research continues to lend substantive theoretical insights to urban planning and regional science activities. A painfully incomplete set of important examples include Evans (1973), Wilson and Senior (1974), Hartwick and Hartwick (1975), Los (1979), Guldman (1979), Hopkins (1979), Brill et al. (1982), and Kim (1986)².

In the context of urban systems, numerical simulation³ models offer an attractive alternative to more traditional analytical approaches. This attractiveness is distinct from numerical modeling's obvious empirical relevance. It is often possible to solve numerical versions of complex, theoretical models from which it would be very difficult to extract closed form solutions. Further, algorithmic solution procedures involved are, by definition, suited to the capabilities of high speed computers.

The availability of high speed computing is not a panacea for the difficulties involved in formulating, solving, and using complicated models; and numerical simulations alone cannot displace the role of more analytical approaches. Rather, the two avenues complement each other, with numerical simulations providing more rapid access to the insights required to achieve more general results. Mills' series of urban activity models (Mills, 1972, 1974a, 1974b, 1975) are our principal examples.

Urban spatial structure is the outcome of a process that allocates activities to sites. The process is principally one of transactions between owners of real estate and those who wish to rent or purchase space for their homes and businesses. These transactions are accomplished by the general rule of the market. The purposes of this paper are to review and assess the significance of Mills' linear programming model of spatial economy, particularly with respect to subsequent models based on the principles and paradigm of the original Mills model, and to describe an extended Mills model that accounts for intertemporal production with pollution control.

**PRECURSOR MATHEMATICAL PROGRAMMING MODELS**

Mathematical programming techniques constitute a particularly useful subset of system simulation tools. Mathematical programming models of urban systems might be linear dis-

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² These citations ignore a large and important literature pertaining to facility location problems. Facility models tend to be mixed integer programs or quadratic assignment problems, and are usually classified as "location" rather than "land use" formulations. Because of the difficulties associated with treating mixed integer constraints, these location problems tend to be analyzed from a mathematical programming rather than an economic perspective. A complete review of location theoretic models can be found in Brandeau and Chiu (1989).

³ "Simulation" does not necessarily imply (but does not exclude) discrete event simulation. Rather, the reference pertains to any numerical representation and treatment of the functional forms defining a given system.
crete, or otherwise nonlinear. Linear programming models offer some of the clearest modeling advantages (and disadvantages). Advantages include uniqueness of the optimal solution, readily accessible market interpretation of primal/dual outputs, and good to excellent speed of solution characteristics. These aspects have been investigated extensively in a number of publications, and this body of knowledge can be directly applied to the formulation and analysis of linear programming models of urban systems. In addition, time staged optimization problems can often be formulated as multiperiod linear programming models.

Mixed integer programs (MIP) and smooth nonlinear formulations such as quadratic programs (QP) are competing alternatives for representing urban systems. Such formulations can be used to treat a number of important circumstances, including indivisible activities, synergistic interactions between decision variables, affine cost functions, and increasing or decreasing returns to scale. Unfortunately, the introduction of nonlinear relationships into an otherwise linear model can decimate the convexity of the feasible region, introducing the possibility of local optima. In addition, the introduction of integer and/or nonlinear variables combinatorially increases solution times, while simultaneously obscuring primal/dual relationships. The models of production and residential land use of Koopmans and Beckmann (1957) and Herbert and Stevens (1960) elegantly demonstrate the applicability of operations research techniques in the context of urban and spatial economic theory.

The Herbert–Stevens model

The Herbert and Stevens (1960) model is the seminal application of linear programming to the study of residential location. The optimal distribution of residential land uses maximizes the total of all profits earned by developers, i.e. the total rent paying capacity of the system. Constraints ensure that the quantity of land consumed in any zone cannot be greater than the area of the zone, and that every household must be located somewhere.

Inspection of the dual formulation reveals that residential land use is optimized when the total of all rents and taxes paid in the system minus all subsidies paid in the system is minimized. The unit rent paid in each zone is never less than the rent paying capacity of each household located there. Though rents must be positive, group specific subsidies may be either positive or negative. The presence of tax/subsidy terms in the dual Herbert–Stevens formulation follows from the fixed population levels and bid rent coefficients in the primal problem. Complete market equilibrium implies that residential sites go to the households bidding the most rent for each site. Exogenous bid rents are adjusted by taxes and subsidies. The implications of these partial equilibrium results have been investigated in several subsequent contributions to the literature. Wheaton (1974) provides an adjustment procedure that drives the tax/subsidy terms to zero. The simplification of the problem by Anderson (1982) demonstrates the equivalence between the adjustments in the tax/subsidy terms and equilibrium adjustments in household utility levels. Pines and Werczberger (1982) demonstrate that the simplest way to drive the tax/subsidy terms to zero is to open the system with respect to migration, and permit population levels to vary optimally.

The principal limitation of the Herbert–Stevens model is that it ignores nonresidential land use. The principal shortcoming of linear programming formulations is that many important activities and processes exhibit discrete and/or otherwise nonlinear characteristics. These nonlinearities include the discreteness of certain facilities, substitutions between factors of production, congestion effects on transportation networks, agglomeration economies, and the spatial attenuation of external effects. Numerous approximations are needed if such relationships are to be treated sensibly in a linear framework. These approximations can significantly increase solution times by increasing problem size.
The Koopmans–Beckmann model

The quadratic assignment problem of Koopmans and Beckmann (1957) incorporates three simplifying assumptions:

1. the transportation network is assumed to be known and fixed;
2. activities are assumed to be indivisible and to interact with fixed, location-independent traffic intensities; and
3. traffic flows experience no congestion costs.

The assumption of indivisible activities is particularly noteworthy, because some degree of physical indivisibility is typical of most production processes. The optimal set of activity location assignments minimizes the system’s total interaction and location costs. Each indivisible activity is assigned to exactly one site, and each site receives exactly one indivisible activity.

Koopmans and Beckmann identified a set of general conditions under which the optimal solution to the smooth version of this quadratic program could not be integer, although it has been shown by Heffley (1976) and others that there also exist special conditions under which integer outputs are optimal. In general, however, the solution to this discrete quadratic programming problem is not price sustainable. This result has made the Koopmans and Beckmann model a less popular tool for investigating the market for urban land, though the literature includes a number of ambitious quadratic programming formulations directed at planning applications. Important examples include the joint land use/network design models put forth by Hopkins (1977), Hopkins and Los (1979), and Los (1978, 1979).

Gordon and Wingo (1981) address the question of price sustainability by noting that the static nature of the Koopmans–Beckmann model attaches too high a degree of simultaneity to the location decisions of competing agents. These simultaneous location decisions have no plausible empirical interpretation in a world in which the decisions of competitors are almost always sequential.

The Beckmann–McGuire–Winsten model

The Beckmann et al. (1956) path flow model of (static) user equilibrium in transport identifies endogenous network link and path costs under conditions of elastic demand. Network equilibrium path costs are important because accessibility is assumed to have an important bearing on any site’s economic attractiveness. Unfortunately, this convex programming formulation is intractable because it implies the pre-enumeration of all network paths. However, the elastic demand problem can be reformulated in terms of link flows, and converted to an equivalent fixed demand model (Gartner, 1980a, 1980b). The link flow version can be stated in a polynomial number of steps, and is subject to iterative solution (LeBlanc et al., 1975); but it does not determine unique path flows. However, user equilibrium conditions imply that the travel time on all used paths between all origins and destinations are equal, and the minimum cost path between any origin and destination must necessarily be an equilibrium cost path. Consequently, optimal link flows can be recovered by solving a polynomial time all shortest paths problem.

MILLS’ LINEAR PROGRAMMING MODEL OF EFFICIENT LAND USE

Mills’ (Mills, 1972, 1974a, 1975) innovative linear programming model of urban land use and transportation defines a highly tractable, economically complete perspective from which to study spatial organization, providing significant theoretical extensions of the LP methodology’s applicability to urban systems modeling.
The Mills model and its closest derivatives (Hartwick & Hartwick, 1974, 1975; Kim, 1978a, 1978b, 1979, 1983) are static, cost-minimizing LP formulations that optimize the spatial distribution of transportation infrastructure, housing, and fixed coefficient production given final demand or export requirements and fixed location export sites. All sectors of the urban economy are accounted for; and all production costs, including transportation and housing costs, are minimized jointly. Prices, population, the complete distribution of land uses by activity and capital intensity, and transportation flows are determined endogenously. Mills’ work and the work of his students provides considerable insight into the static version of efficient urban form.

All imports and exports are assumed to enter and exit the urban system through predetermined export sites. Land use zones are differentiated by their position on the plane, and by distance from the export node. Land is the only primary factor of production in short supply. Labor and capital are assumed to be available in unlimited quantities at fixed wage and interest rates. The linear programming tableau defining a Mills model or derivative formulation optimizes the urban system’s land use configuration by minimizing the sum of production and transportation costs subject to the following constraints:

1. minimum export requirements must be met;
2. flows of goods and labor into and out of zones must be conserved subject to zone specific production and export activities;
3. zone specific land availability limits cannot be exceeded;
4. transportation services provided must be sufficient to meet the demand for service; and
5. the primal variables must be nonnegative.

Given $E$, a vector of the total quantity of goods to be exported from an urban area; $F$, total household final demand; and $A$, an input–output matrix, the problem is to find the optimal set of activity locations and commodity flows for which the total costs for producing $X$ are minimized. These include the costs of transporting intermediate and final goods from origins to destinations, and export (and import) costs.

Mills specification makes the following three significant contributions.

1. The location and extent of urban activities can be found as the result of the simultaneous demand for and supply of land uses and transportation.
2. The intensity of land uses can be determined by the introduction of three dimensional input–output coefficients, $a_{qs}$, that represent the amount of input $q$ required per unit output $r$ at land use intensity level $s$. The three dimensional input–output coefficients are defined such that when $q$ is the land input, the quantity of $q$ required to produce one unit of output $r$ decreases as $s$ increases. The magnitude of these shifts varies across commodities. Similarly, the capital inputs required to produce one unit of $r$ increase as $s$ increases. These shifts vary across commodities.
3. Export base models have been operationalized. The Mills model includes an assumption that an urban area exists because it exports goods. An urban area’s economic activities are an explicit function of the total export levels achieved by the city. This assumption is realized by imposing the condition that all export goods will have to be exported through predetermined export nodes such as central business districts (CBD), through suburban centers, or through the periphery.

In addition, Mills demonstrates that many of the nonlinear relationships characterizing urban systems can be meaningfully approximated in the context of linear programming. Transportation costs can be represented by a linearized, step wise measure of congestion, in which case the constraint set also includes logical constraints requiring that transportation facilities must exist before they can be congested.
EXTENSIONS OF THE MILLS MODEL: STATIC FORMULATIONS

There are many important problems in spatial problem analysis, but we submit that the following are paramount.

1. In a mixed economic system, both public and private sectors play important roles in shaping spatial phenomena.

2. Spatial equilibria such as the extent and location of land use and transportation activities are the results of both simultaneous and sequential interactions between various actors. Most existing spatial conditions have been established incrementally.

3. The single most important characteristic that distinguishes urban area from rural areas is intensive use of land that results in high rise buildings, pollution, and transportation congestion. The goal of urban planning is not the elimination of these externalities, but an efficient allocation of resources in light of these effects.

Mills' model opens the door to address the third point; and, to a lesser extent, the second. Subsequent formulations address the first.

Linear programming approaches

One of the first extensions of Mills models had to do with the realistic treatment of urban areas. Hartwick and Hartwick (1974) extend Mills' original model by treating a multinucleated urban center in conjunction with intermediate production goods. Kim (1978a, 1978b), Kim (1979) provides a sector based extension which includes a multimodal transportation system requiring the specification of network contiguity constraints.

Nonlinear programming approaches

While the supply of transportation services is determined simultaneously with other urban land uses in the basic Mills model (Mills, 1972, 1974a, 1975), transportation requirements are determined only with respect to land and capital costs. From this perspective, the Mills model can be considered a simple network design model. The model presented in Mills (1974b) considered congestion within an integer programming framework, but it is still a highly abstract representation. If one applies the Mills model in an attempt to determine an optimal expansion of an existing urban area, the schematic treatment of road networks may produce a simplistic (and thus inefficient) solution with respect to the transportation network.

More recently, Kim (1983, 1986) and Kim and Rho (1988) coupled nonlinear congestion costs from the Beckmann–McGuire–Winsten (Beckmann et al., 1956) model with minimum entropy constraints, defining a path for the analysis of optimal urban expansion. This model was applied to the city of Chicago, identifying the optimal activity locations associated with competing investments in transportation infrastructure. The most recent application of this model is by Linqvist et al. (1992) in a comparative study of Chicago, Seoul, and Stockholm. Though this is a sophisticated application of Mills' approach in a nonlinear programming framework, it remains essentially a static exercise. Such static formulations cannot be used to model a sequence of changes in the state of an urban activity system.

Regional models

Transportation is an important public service, and a principal determinant of spatial economic activities. The location and means of production and the origins and destinations of movements of people and goods are affected by the provision of transportation facilities. At the same time, the location of private activities affects the demands imposed on the trans-
portation system. It matters a great deal how much and where transportation is provided in a spatial economic system. However, most programming models, including the Mills' extensions, assume that the goals of the public sector and private sectors are congruent.

In fact, these goals are not usually the same. The public sector's goal is to enhance the welfare of the public, while the private sector usually pursues its parochial interests. These different and sometimes conflicting goals have been modeled in bi-level programming frameworks by Kim (1989, 1990), and Suh and Kim (1989, 1992). Under this approach, two different objective functions are provided. At the upper level, the public sector's welfare is maximized with respect to the government's budget constraint, and the requirement that import and export targets be met. At the lower level, the private sector minimizes production and transportation costs subject to the usual constraints, including conservation of flows and spatial interactions. Unfortunately, solutions to bi-level programming models can be difficult to obtain and may be nonunique. Anandalingam and Friesz (1992) identify a number of improved solution algorithms for bi-level models.

Kim et al. (1983, 1985) developed a combined input–output and commodity flow model in a nonlinear programming framework. Kim and Kim (1985) formulated a national transportation development planning model in a linear programming framework to find an optimal highway network for Korea. These models include nonlinear congestion functions and an entropy constraint that preserves a minimum level of spatial interaction among regions, thus overcoming the tendency for linear programming models to identify only diagonal interregional trip matrices.

Lastly, the notion that an urban area's economic activities depend on export requirements provides a logical extension of the Mills model to the analysis of a nation's spatial development, particularly developing countries whose national development goals are based on export promotion strategies. A series of these models have been built as extensions of Mills' original model. These extended models are summarized in a recent book by Kim (1990).

EXTENSIONS OF THE MILLS MODEL: DYNAMIC FORMULATIONS

The Mills model and its derivatives have exogenous centers, but contemporary metropolitan areas are characterized by decentralized patterns of employment. As a large metropolitan region grows, a point is reached at which economic activity and population begin to relocate from the metropolitan center to subcenters. The emergence, growth, decline, and obsolescence of individual urban subcenters is part of a dynamic process resulting from simple economic behavior. Some authors characterize this evolving form as a counterurbanization that implies erosion of a single centered metropolis, and as a process of population deconcentration characterized by decreasing densities and increasing local homogeneity (Berry, 1976). Others describe it as a dispersion of activities producing a random sprawl of tract housing, shopping malls, and industrial parks, each locating without any specific relation to particular focal points (Blumenfeld, 1964). Many see this emerging form as a mixed blessing, increasing the consumption of land and other finite resources (Clark, 1954). Regardless of the perspective taken, the study of how such subcenters develop and their impacts on land values, population distribution, and travel patterns is highly relevant to the investigation of land use and transportation interaction.

Empirical studies abound. In 1980, 57% of all office space in the US was located in urban centers and 43% in suburbs; by 1986, 60% was in the suburbs, compared to 40% in the centers (Pisarski, 1987). Originating in America's sunbelt (Cervero, 1986), the suburban office boom has become nationwide, occurring even in older industrial areas. In greater
Mills' urban system models: perspective and template for LUTE applications

Philadelphia and St. Louis, for example, suburban employment grew by 8 and 17%, respectively, between 1982 and 1986, contrasted by a loss in central city jobs over the same period (Orska, 1986; Urban Land Institute, 1987). Erickson (1983) describes the evolution of suburban economic activities in the US through 1960 as a process of "dispersal/differentiation", followed by a subsequent phase of "infilling/multinucleation". Hartshorn and Muller (1986) describe the changing of land uses in American suburbs in terms of four stages, including bedroom communities (pre-1960), independence (1960–70), catalytic growth (1970–80), and high rise / high technology (post-1980).

Many expect the trend toward decentralization to accelerate in the coming years as America's economy continues to shift from a manufacturing base to an emphasis on service and information processing activities. Suburban areas offer cheaper land, reduced externalities, proximity to regional airports, smart buildings laced with fiber optic cables and advanced telecommunications equipment, pools of second wage earners, and country like amenities (Dowall, 1987; Urban Land Institute, 1986, 1987).

Relevant specifications of urban space must represent conditions under which policentrism might emerge, discussing where the centers may be located (Richardson, 1988). Blackley and Follain (1987) argue that accessibility to amenities other than workplaces need to be accounted for in locational equilibria. This implies that spatial externalities and time should both be represented in location decisions. Urban systems are subject to temporal changes that static, long run economic models cannot accommodate. Even under contrived conditions, it is inappropriate to treat urban form as the product of a steady state process. Urban land use and capital configurations are the products of continuous technological, economic, and social change. Unfortunately, closed form models of spatial production systems are difficult to formulate in a dynamic context, and much more difficult to solve than their static counterparts.

**Intertemporal linear programming approaches**

The intertemporal extension of Moore and Wiggins (1990) endogenizes export levels to optimize a profit maximizing Mills model over time. Capital is replaceable, and optimal changes in the structure of land use are executed with finite cost. These extensions identify and explain the differences between long run and static economic equilibriums.

It is tempting to presume that the optimal configurations of static urban systems usually conform to long run economic results. Any system insulated from exogenous shocks might be casually assumed to eventually achieve an efficient (static) configuration independent of the system's initial state. However, it is simple to see that this is not the case. Long run systems replicate static results only if undisturbed throughout their entire history.Transient changes in economic or technical conditions produce long run economic equilibria that, while efficient in a dynamic sense, appear to be very suboptimal if interpreted from the static point of view. For example, introducing dynamics makes it possible to identify conditions under which reverse commutes will occur in a perfect market for urban land and transportation (Moore & Seo, 1991), an outcome too widely assumed to be impossible.

**Sequential programming approaches**

Gordon and Moore combine these results with the work of Gordon and Wingo (1981) to undertake a theoretical and computational investigation of a dynamic land use transport model that includes discrete activities and proximity dependent externalities, including network congestion (Gordon & Moore, 1989, 1991; Moore & Gordon, 1990). This work resolves some of the standing questions concerning quadratic assignment problems and what
the solutions to such problems explain about the market for urban land.

Gordon and Moore divide the urban area into many discrete sites. These sites have different attributes. Each site belongs to an owner who is free to sell or lease his property. At the beginning of each transaction period, every establishment evaluates the merits of every site, and decides what price it would be willing to pay for access to each site. The passage of time brings changes in the number and types of establishments bidding for access to locations. Existing establishments also change in terms of their characteristics. Households change in size, manufacturers acquire new production methods, and retailers shift product lines. Some sites change hands and some establishments move to new locations. As long as some establishments are moving, the pattern of accessibility and contiguity changes for other establishments. Even if site characteristics are fixed, these various changes accumulate over time to cause significant shifts in the matrix of site bids.

Locators are assumed to make decisions from a ceteris paribus perspective (Moore & Gordon, 1990). By solving a series of linear assignment problems that track urban land use over time, their model presents a sequence of urban location decisions resulting from locators' efforts to maximize net revenues by mitigating congestion costs and other externalities. Network congestion and other effects are endogenous in each period, but traffic intensities between all activities are exogenous. No centralized control strategies are represented: locators merely compete, accounting for the external effects inflicted by others but not by themselves.

The nonlinear assignment problems conventionally used to represent such systems can be separated into a sequence of smooth nonlinear programming problems and linear assignment problems. The arrival, departure, and ongoing bidding of activities constitute the principal mechanisms for spatial rearrangement. Unsuccessful bidders are consigned to a null site, or queue. Activities bid nothing for access to the queue, and there is no constraint on the number of activities that can locate there. To represent this process in a more complete way, Moore and Gordon also introduce a null activity called 'vacancy' that bids nothing for physical sites and can be assigned to any number of sites. When nonvacancy activities offer (sufficiently) positive bids for sites, existing vacancies are displaced.

This extension simultaneously delivers realism, tractability, and accessible dual formulations à la Mills. This approach explains transactions in the market for urban land in terms relevant to the objectives of the participants and external authorities. In addition, this body of work reveals that zoning schemes affect not only the rents accruing to land owners, but to the owners of capital as well (Moore, 1991). This result is unanticipated in the transferable development rights literature, most of which focuses on windfalls and wipeouts in the market for urban land.

**Regional modeling approaches**

One of the least satisfactory features of regional analysis is the gulf between the studies of regional economic change and the study of regional spatial structure. Recent regional economic analysis concerns empirical and theoretical developments in growth theory, econometric modeling, and input–output techniques, but are rarely concerned with spatial structure. Similarly, studies of spatial structure are generally undertaken in a static context seldom related to the process of regional economic change. In fact, contemporary suburbs are interdependent, collectively comprising the metropolitan economy. This metropolitan economy is, in turn, part of a larger system of economies, engaging in trade with its hinterland, other metropolitan economies, and the rest of the world. At the same time, the metropolitan region is an economy with an evolving differentiation between suburbs.

Interdependence between activities is an important factor in the growth of regions. Inter-
actions between agents makes the location decision of one agent dependent on the location decisions of other agents. Input–output relationships are important determinants of clustering both within and between activities. Interdependency is further influenced by standard structural transformations in the composition of demand, trade, production, and factor use in a developing economy (Chenery, 1960).

Recognizing interdependencies is the principal means of integrating the regional economic development process into an activity location model. Interactivity flows are conditioned on economic development patterns that include changes in the composition of demand, trade, production, and factor use as functions of per capita income. Seo et al. (1995) contend that the process of metropolitan economic growth drives transformations in the spatial structure of the activity system. Their study depicts the dynamics of land use patterns, integrating Chenery’s regional economic development processes into an activity location model. Structural transformations are revealed by nonproportional growth across sectors. Economic development produces changes in input–output relationships that are translated into updated transshipments between activities.

The Seo, Mocro and Gordon model is a simulation that accounts for interactions between:

1. *a priori* profitabilities;
2. transport costs defined by a congestive transportation network;
3. externalities;
4. relocation costs; and
5. technological change.

This sequential urban land use model consists of two major components:

1. a discrete programming model of the market for urban land and transportation; and
2. an interactivity flow system that accounts for structural transformations resulting from economic development.

The model is initialized by an exogenous economic structure and spatial pattern. Given this initial spatial pattern, the characteristics of establishments change. Given income elasticities for each sector and an existing set of input–output relationships, the structural transformation model endogenizes production levels and traffic intensities. The discrete programming model identifies a new land use pattern. In the next time period, more structural economic changes are realized and the structural transformation model once again produces new traffic intensities. Taken together, these factors tractably explain the evolution of an urban economy, and the effect of this evolution on urban structure. The approach is summarized in Fig. 1.

**GULDMANN’S INTERTEMPORAL MODEL OF POLLUTION CONTROL**

The two most important urban externalities are congestion and pollution. The external costs of these effects are large, and their structure can be partially deduced from an engineering representation of transportation and production. Neither the original Mills model nor its immediate derivatives treat pollution externalities, though the most recent efforts of Kim (1989) handle congestion well enough that his Chicago model serves as an effective template for policy analysis. The Gordon and Moore (Gordon & Moore, 1989, 1991; Moore & Gordon, 1990) and Seo et al. (1995) models include externalities, but only as part of an accounting framework. Locators react to externalities, but no control strategies are addressed.

Guldmann (1979) provides a multiperiod, mixed integer programming model of land use
and pollution control that incorporates a matrix of interzonal pollution transfer coefficients. The model optimizes production locations based on zone specific pollution and pollution abatement costs. This intertemporal formulation is an extension of his previous work with Shefer (Guldmann & Shefer, 1977; Shefer & Guldmann, 1975) on static models of optimal pollution control. In the most general of Guldmann’s intertemporal formulations, pollution costs, pollution abatement costs, development costs, facility relocation costs, and commuting costs are minimized jointly over time subject to various (mostly aspatial) constraints. These include a number of relationships ensuring the endogenous determination of zero-one decision variables relating to air quality and facility location priorities. These numerous integer variables make Guldmann’s formulation difficult to optimize under all but trivial conditions.
With minor modifications, a smooth version of Guldmann's formulation constitutes a general equilibrium model of the spatial market for pollution and pollution control. However, Guldmann's (Guldmann 1979) optimization efforts have been restricted to partial equilibrium applications in which the majority of the formulation's decision variables are replaced with exogenous assumptions. A fixed number of production and residential facilities are located sequentially according to a predetermined schedule. Residential and industrial zones are differentiated a priori, and transportation variables are ignored.

In contrast to most of the Mills models, the Guldmann (1979) model is a dynamic, mixed integer programming (MIP) formulation that optimizes the locations of indivisible production facilities given zone specific costs of pollution, pollution abatement, and facility location change. In the case of previous linear programming formulations (Shefer & Guldmann; 1975, Guldmann & Shefer, 1977), production facility locations are exogenous. In contrast, the production facility locations in Guldmann's intertemporal model are endogenous, represented by zero-one decision variables. Significantly, the total number of production facilities located and the location schedule remain exogenous.

FURTHER EXTENSION: AN INTERTEMPORAL LUTE MODEL OF PRODUCTION, TRANSPORTATION, AND POLLUTION CONTROL

Structure of an intertemporal LUTE model

Mills' original formulation has been synthesized with other approaches that emphasize both theory and application, producing methodological advances in both dimensions. The shift toward policy oriented applications is particularly useful. Transportation and location cannot be studied in a meaningful way without an understanding of relevant public policy. Market based interventions such as transferable emissions rights, transferable development rights, and road pricing are particularly amenable to extensions of the Mills approach.

Given sufficient structural assumptions, Guldmann's (1979) representation of pollution processes and costs could be restated in the context of Moore and Wiggins' (1990) dynamic extension. These two classes of models are based on assumptions that are, for the most part, consistent. Much of the information Guldmann's models require as inputs appear as decision variables in the Mills models. By restating Guldmann's formulation in terms of Mills', it is possible to specify a computable model of the urban activity system that endogenizes both production and pollution processes in an intertemporal framework. This research model is internally consistent in terms of information, assumptions, and theory; captures the important economic and technical determinants of land use; and is unique in its capacity to capture trade-offs between production, environmental quality, and infrastructure investment decisions.

The objective of developing such a model is to define a tractable framework that will treat optimal pollution abatement strategies in the broader context of Mills' general equilibrium formulation. Such strategies include command and control regulations, emissions and development taxes, and the definition of well organized transferable development rights (TDR) and transferable emissions rights (TER) markets.

Using the dynamic Mills model as a template for improving the general equilibrium characteristics of Guldmann's intertemporal model, a new formulation can be achieved in which production levels, the spatial and temporal distributions of activities and structures, interzonal travel, pollution levels, and pollution abatement levels are all endogenous; and the benefits of market interventions are explicit.
Profit maximization

Exercising the dynamic Mills–Guldmann model maximizes the present value of the urban system's profit stream. In every period, gross revenues are computed as the product of exogenous prices and endogenous export levels. Endogenously determined costs include:

1. the total cost of the land, capital, and labor inputs needed to support all production, storage, transportation, and residential land uses;
2. the total cost of food and fuel imports;
3. transportation line haul costs;
4. construction and reclamation costs; and
5. pollution and pollution abatement costs.

Period specific profits are discounted at an interest rate equal to the exogenous rent on capital. The boundary period at the end of the planning horizon is defined by long term forecasts or targets.

Production

Production is described by alternative, fixed coefficient technologies allowing discrete trade-offs between production inputs. Fuel is one such input. Some production activities are basic, i.e. export oriented, but others are associated with the intraurban demands generated by residents. Production and associated transportation flows result in either case. In the interest of computability, production activities are divisible, represented by linear rather than integer variables. A subsequent attempt will be made to extend the formulation by treating these activities in a discrete way. Zone specific pollution abatement levels are linked to zone specific production levels and associated fuel burning. Production levels are endogenously determined for each commodity, technology, location, and time period.

Storage

Period specific production is restricted to exports and associated requirements for intermediate production goods. It is possible to store both intermediate production goods and export goods indefinitely, allowing for inventory strategies.

Capital replacement, maintenance, and land use change

Construction and reclamation activities are defined to account for the cost of reconfiguring land uses. In contrast to the Mills model, capital investments corresponding to structures are translated into physical measures. By identifying these entities with a quality measure, optimal maintenance schedules can be derived. The 'capital' label is reserved for perfectly mobile investments, and the cost of transporting materials related to construction and reclamation activities is explicit. Unlike Guldmann's interperiod model, there are no boolean constraints requiring previously developed land to be redeveloped as quickly as possible. Consequently, the combined model dispenses with a large number of integer variables at no cost to generality.

Transportation

The endogenous level of transportation service provided must be sufficient to meet the demands for service imposed by:

1. import and export commodity flows;
2. flows of intermediate production goods, including nonbasic goods;
3. construction materials and salvage flows from reclamation activities; and
4. labor flows.
As a first approximation, congestion costs are ignored to avoid the introduction of integer or other nonlinear variables, and to conserve linear variables. A more sophisticated treatment of congestion is necessary and will be pursued subsequently.

**Pollution**

The Schefer and Guldmann (Shefer & Guldmann, 1975) pollution transfer matrix is used to describe the transfer of airborne pollutants emitted by zone specific sources. Pollution costs are traded-off against abatement costs in the manner suggested by Guldmann, but these pollutants are treated as one of the costs associated with endogenous production and residential location decisions.

Zone specific limits on total pollution levels consist of one of two alternative types:

1. a linear constraint recognizing the maximum carrying capacity of uninhabited zones;

and

2. an integer defined boolean constraint imposing a lower ceiling on acceptable pollution levels in the event of residential land use in a given zone.

Because only one such integer constraint is required per zone, the number of integer variables imposed by the boolean constraint does not inhibit solution.

Alternatively, zone specific pollution constraints can be linearly linked to residential land use. This displaces integer variables and is consistent with the form of some existing interventions, but also raises equity questions unless urban residents are compensated for the incremental health risks generated by their increased exposure to contaminants. Also, pollution effects can be made cumulative over both time and space. Period and zone specific pollution constraints can be adjusted based on pollution levels realized during preceding time periods. For example, if pollution constraints are binding in period \( t \), it might be desirable to impose stricter, rehabilitative requirements on the system during period \( t+1 \).

**Applications of an intertemporal LUTE model**

A dynamic Mills–Guldmann model that realistically incorporates urban externalities might be used to analyze a number of different policy questions. Data permitting, this analysis could be predictive, prescriptive, or both. Alternatively, such a model might be used solely to investigate duality relationships and their implications for general equilibrium solutions in which external effects are explicit.

**Activity shifts in the urban land use system**

The logical role for the outputs of such a model is as an administrative template for guiding land use change. In addition, this approach permits the cost of suboptimal land use alternatives to be assessed in the event that such alternatives were found to merit consideration based on criteria unaccounted for in the model. For example, the present value efficiency cost of such instruments as zoning variances can be identified.

**Optimal pollution, pollution control, and treatment infrastructure**

By accounting for pollution and pollution control costs in a dynamic, general equilibrium model, optimal pollution control policies can be determined in conjunction with optimal land use and production schedules. The efficiency costs and benefits of zone specific pollution standards can be assessed, as can the present value costs and benefits of temporarily relaxing such constraints. Land use shifts associated with new pollution controls on vehicles and point sources can be identified. The financial viability of new public infrastructure for pollution abatement can be determined under a wide range of different technical and eco-
ocious assumptions. The information provided by the associated dual formulation permits pollution taxes and abatement subsidies to be optimized on a period specific basis, or might be used to structure a Transferable Emissions Rights (TER) market.

**Employment**

A dynamic Mills–Guldmann model makes it possible to assess both the economic impact of pollution abatement on the location decisions of firms and households, and the welfare implications of regulatory and/or compensatory requirements.

**Health risk management**

Zone specific pollution limits could be established based on both intensity and duration of exposure to contaminants. The cost of violating zone specific pollution standards could logically be tied to the risk exposure of the affected population. This risk exposure has a temporal component, and firms might be assessed for the costs of violations based both on the intensity and duration of the violations involved. Assessments might then be used as insurance premiums to mitigate the risk experienced by affected residents.

**Theoretical considerations**

As defined above, the intertemporal LUTE model is a smooth, linear formulation. We ignore agglomeration economies, and if pressed choose to classify and treat them as extern- nalities. However, there remain at least two other important sources of nonlinearity to be addressed. These are congestion effects, and information constraints.

Congestion effects might introduced via step functions or other: piecewise linear approximations, or by replacing transportation cost coefficients in the objective function with nonlinear congestion functions. If these functions are sufficiently well behaved, the modest nonlinearities that result can still be addressed by robust computational procedures (Boyce, 1990; Boyce & Lundqvist, 1987). No standard formulation for addressing dynamic network equilibrium problems has emerged, and those currently described in the literature are not particularly tractable. Consequently, any representation of congestion would almost certainly have to be static within time periods.

The role of entropy in this context is more difficult to address. In a static model, a minimum entropy requirement might be appended to the constraint set as a means of recognizing the bounded information and rationality of the actors represented in the model. The attendant problems, computational and otherwise, are nontrivial, but a worthy investment from both theoretical and empirical perspectives. Recognizing and imposing an appropriate entropy level is central to calibration activities.

In an intertemporal context, however, entropy requirements are more difficult to interpret. Entropy is essentially a static construct, and it is possible to inadvertently misinterpret deterministic adjustments in a dynamic system as static randomness. A dynamic, deterministic system may appear disordered and suboptimal if measured in static terms. What component of an urban system’s apparent disorder is due to uncertainty, and what part is actually a rational, structural response to changing conditions? Measuring the entropy of a dynamic system at a single point in time collapses static disorder and dynamic adjustments into a single quantity. From a static perspective, temporal adjustment and expectations may be just as legitimate a source of disorder as bounded information. If so, then whether entropy measures reflect a random static choice or a series of deterministic choices is irrelevant. However, we have never trusted our luck, and suspect the situation is more complicated. It is probably theoretically inconsistent to collapse intertemporal order into a static measure of disorder.
This question of how much disorder is random and how much is actually a form of dynamic order is visited on any temporal economic system, LUTE or otherwise. The unfortunate truth is that both sources of apparent disorder tend to be present in any behavioral systems. Thus, it is appealing to think in terms of period specific equilibria, recognizing that there are (changing) information constraints imposed on the actors whose decisions generate these equilibria. We have chosen to be conservative. The dynamic LUTE model is as complicated as any we have described or applied. Much work remains to be done on and with it, particularly with respect to primal/dual relationships. Consequently, we have continued to emulate Mills’ approach and, at least in case of this theoretical exercise, trade off some structural realism in favor of a simpler, i.e. linear approach. Time and progress will tell how well this abstraction serves.

REFERENCES


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Mills' urban system models: perspective and template for LUTE applications
