REVERSE COMMUTATION IN A MONOCENTRIC CITY*

James Elliott Moore II and Jong-Gook Seo
School of Urban and Regional Planning and Department of Civil Engineering, University of Southern California, University Park, Los Angeles, CA 90089-0042

ABSTRACT. A general equilibrium multiperiod linear programming model of urban land use is used to identify reverse commutation as a rational response to economic change. Information about future economic conditions is complete, and capital is replaceable at finite cost. Export mixes are defined exogenously, and temporal shifts between activities of different land intensities are shown to induce reverse commutes as one way of avoiding the cost of land use change.

1. INTRODUCTION

The paper presents a dynamic linear programming model of the urban economy that identifies reverse commutation as an optimal economic strategy under reasonably general conditions of economic change. A numerical version of this unified, general equilibrium model is specified and optimized, and a summary of the outputs explained. This multiperiod linear program belongs to a class of formulations originated by Mills (1972, 1974a, 1974b, 1975) and extended by others (Hartwick and Hartwick, 1974, 1975; Kim, 1978a, 1978b, 1979, 1986; Moore, 1986; Moore and Wiggins, 1988, 1990; and Rho and Kim, 1989). These precursors are collectively identified here as "Mills heritage models."

Static vs. Dynamic Modeling Perspectives

The investigation of static systems in long-run equilibrium has contributed much to the foundations of urban economic theory. The implicit assumption underlying time independent models is that, given a sufficient period of freedom from environmental shocks, any economic system will achieve an efficient configuration. In a monocentric city, this efficient configuration conventionally includes contiguous land uses (Ohls and Pines, 1975), uniform land use at locations a fixed distance from the center of the city (Mills, 1981), and the absence of reverse commuting patterns (Mills, 1972). As indicated in Figure 1, reverse commutation occurs when labor travels from residential sites located at the urban interior to production sites located near the urban periphery. The transport costs associated

*The authors gratefully acknowledge useful comments and contributions from Alex Anas (Northwestern). We received computational assistance from Geun-Young Kim and Jong-Gyu Lee, both of whom are Ph.D. candidates in the School of Urban and Regional Planning, University of Southern California. We are particularly grateful to Ken Small (University of California at Irvine), who announced to a USC seminar audience that reverse commutation is inevitably inefficient in monocentric urban systems. The usual caveat applies.

Received July 1990; revised March 1991, accepted April 1991.
with any static spatial arrangement involving reverse commutation can be reduced if the locations of residential and production activities are exchanged. It follows that reverse commutes are suboptimal and would not occur in the context of a perfect market for urban land and services. From a static perspective, the suboptimality of reverse commutes is a reasonable conclusion. Nevertheless, the economic and technical environments in which urban systems exist are subject to changes that static models cannot capture. It is inappropriate to treat urban form as the product of a steady state process, nor is it necessarily true that steady state conditions will propel an existing urban system toward a static equilibrium (Moore and Wiggins, 1990).

A Linear Programming Example of the Static Case

The suboptimality of reverse commutation in the static case can easily be verified with a small linear program. The simplest nontrivial model is premised on a pair of embedded, annular zones surrounding a central export node. Transportation is assumed to be ubiquitous. Intrazonal transport costs are zero. The mathematical specification of the model follows.
\[ X_r(i) = \text{the production of commodity } r \text{ in zone } (i) \text{ (endogenous);} \]
\[ F_r(i) = \text{the flow of commodity } r \text{ out of zone } (i) \text{ (endogenous);} \]
\[ X_r^-(i) = \text{the quantity of the export commodity } r \text{ exported out of the urban system from an export node located in zone } (i) \text{ (exogenous);} \]
\[ A(i) = \text{the area of zone } (i) \text{ (exogenous);} \]
\[ a_{q,r} = \text{the quantity of production input } q \text{ required per unit of commodity } r \text{ produced, with } q = 1 \text{ denoting land, } q = 2 \text{ denoting capital, } q = 3 \text{ denoting the export commodity (exogenous); and} \]
\[ f_r = \text{the unit transportation cost of moving 1 unit of commodity } r \text{ across the boundary between adjacent zones (exogenous).} \]

Synthetic values for the exogenous variables on this list are summarized in Table 1.

**Two-commodity Two-zone Formulation**

The simplest static model locates production and residential land uses to minimize total transport costs subject to constraints enforcing conservation of flow, land availability, and minimum export requirements. Applying the synthetic parameter values, the relevant linear program is

\[
\text{Min } Z = 0.7F_2(1) + 0.7F_3(2) + F_3(1) + F_3(2)
\]

subject to

\[
-F_3(1) + F_3(2) - 0.1X_2(1) + X_3(1) \geq 4.8
\]

\[
+F_3(1) - F_3(2) - 0.1X_2(2) + X_3(2) = 0.0
\]

\[
-F_3(1) + F_3(2) + X_3(1) - 0.5X_3(1) = 0.0
\]

\[
F_3(1) - F_3(2) + X_3(2) - 0.5X_3(2) = 0.0
\]

\[
X_2(1) + X_3(1) \leq 3.0
\]

\[
X_2(2) + X_3(2) \leq 5.0
\]

\[
F_3(1), F_3(2), F_3(1), F_3(2), X_2(1), X_2(2), X_3(1), X_3(2) \geq 0.0
\]

<table>
<thead>
<tr>
<th>TABLE 1: Technical Coefficients</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image" alt="Table" /></td>
</tr>
</tbody>
</table>

| \(a_{q,r}\) | Outputs |
|---|---|---|
| Inputs | \(r = 2\) | \(r = 3\) |
| \(q = 1\) | 1.000 | 1.000 |
| \(q = 2\) | 0.000 | 0.500 |
| \(q = 3\) | 0.100 | 0.000 |

\(f_2 = 0.7, f_3 = 1.0\)

\(X_2^-(1) = 4.8, A(1) = 3.0\) (the central export zone)

\(A(2) = 5.0\) (the urban periphery)
Optimization results are summarized in Figure 2. The first set of results identify the forward commuting pattern associated with static formulations. Production land uses are located closer to the export node than are residences, and labor commutes toward the center. If \( f_p \), the unit cost of transporting labor, is sufficiently increased; then interzonal labor trips cease. Jobs and housing are balanced within zones, and only intrazonal trips occur. The last set of results in Figure 2 was generated by appending the additional constraint

\[
X_i(2) = 0
\]

which prohibits residential land use in the peripheral zone and imposes reverse commutes. The result is a 117 percent increase in total transportation costs.

*Reverse Commutation in the Dynamic Case*

If the relative profitabilities of urban production activities change sufficiently over time, then urban exports will also change, and optimal production strategies may include land use changes. Assuming that shifts in economic incentives do not occur too quickly for the production system to respond, these changes can lead to a number of results that are difficult to replicate in a static context. These results include discontinuous land-use configurations and reverse commuting patterns.

Consider two export-oriented land uses, one capital-intensive and the other land-intensive. Assume that export incentives initially favor production of the capital-intensive good, but then change to favor production of the land-intensive good. In a dynamic, monocentric system, efficiency may dictate that land located at the urban interior either be held in reserve or redeveloped to accommodate subsequent increases in land-intensive production (Ohls and Pines, 1975; Moore and Wiggins, 1990). These vacant sites may be located further from the center of the city than residential sites used by workers initially employed by the capital-intensive production activity. Subsequent labor flows associated with some of the new land-intensive production facilities would then be directed toward the urban periphery rather than toward the center of the city.

The outcome is summarized in Figure 3. In this case the capital-intensive production facilities associated with period 1 are assumed to have been demolished and replaced with the labor-intensive production facilities of period 2. More generally, the capital-intensive facilities might merely be abandoned, in which case reverse commutation would persist. The alternative to this scenario is to develop land in the contiguous pattern suggested by static models, which may lead to increased transportation or redevelopment costs when production incentives change. If these costs are greater than the opportunity cost of holding urban land in reserve, then the optimal land-use pattern will be discontinuous, and reverse commutes may occur.

Viewed from this perspective, reverse commutes are seen to occur in one of two ways. Reverse commutation may be temporarily efficient because discounted commuting cost savings in early periods may be offset by discounted demolition costs arising in later periods when the land-use arrangements that permit inexpensive commuting are reversed. More interestingly, it may be efficient for reverse commutation to persist forever. If the exogenous variables of the urban
FIGURE 2: A (Static) Linear Programming Comparison of Efficient and Reverse Commutation: Values in Boxes Are Areas. (Values Associated with Arrows Are Flows of Goods and Labor.)
FIGURE 3: A Simple Version of the (Dynamic) Efficiency Gain Implied by Reverse Commutation.

economy converge to a stationary state and stay there over an infinite time horizon, then discounted demolition and reconstruction costs may exceed the discounted commuting cost savings realized by alleviating reverse commutes.

2. MODEL STRUCTURE AND ASSUMPTIONS

The qualitative argument for reverse commutation can be verified by optimizing a dynamic Mills heritage model of urban land use. This multiperiod linear programming model is formulated with parameter values conforming to the
assumptions noted above. Land- and capital-intensive exports are initialized at the same level, but exports of the land-intensive commodity increase over time while exports of the capital-intensive commodity decrease.

Overview

The Mills heritage models optimize land uses and capital investments. The following characteristics apply to both static and dynamic versions of Mills’s model.

1. Export goods must be transported from zone-specific production sites to any one of several exogenously located import/export nodes.
2. Imports must be transported from import/export nodes to production sites.
3. Labor must be transported from residential sites to production locations. Land is subject to zone-specific availabilities.
4. Provisions of residential, production, and transportation facilities consumes resources.
5. Population levels are endogenous.
6. Household consumption levels are assumed fixed; and households are assumed to maximize utility by engaging in cost-minimizing behavior.
7. Linear programming models of production processes imply constant returns to scale.
8. Land and capital are primary inputs to the urban economy. Capital is assumed to be ubiquitously available at a fixed price. Land is the only scarce resource. Consequently, all profits from production ultimately accrue to land owners as rents.
9. Finally, given that a Pareto optimal allocation of resources is implied by equilibrium in a perfectly competitive market, it follows that the optimal solution to a Mills heritage model can be sustained by the operation of a perfect market in which prices correspond to the optimal values of the model’s dual variables.

Additional questions must be addressed to complete a dynamic formulation. Two particularly important ones are perfect foresight and replaceable capital. Time periods are optimized simultaneously, and capital investments are assumed to last for an indefinitely long period of time unless replaced at some finite cost (Wheaton, 1982). If land uses are reconfigured over time, resources must be expended to reclaim land from existing uses and dedicate it to new purposes.

Unlike the previous Mills heritage formulations, this version of the dynamic model treats substitutions between land and capital inputs in no more detail than is afforded other factors of production. Production activities are each associated with exactly one technology. The model includes one transportation technology corresponding to one transportation mode.

The hypothetical urban economy has two export commodities: a land-intensive good and a capital-intensive good. The outputs of these activities are mutual inputs. In addition, each activity requires inputs of labor, land, and capital. Technical coefficients are summarized in the Appendix. Labor is produced in the sense that it has group-specific consumption characteristics. Labor’s consumption includes land and capital. The land input satisfies residential site requirements.
Part of the capital input represents the residential dwelling, and part corresponds to the group specific reservation wage available in the agricultural sector. Each unit of labor is assumed to represent a single household.

The demolition activity clears developed land of buildings and infrastructure. Once a parcel has been cleared, the appropriate construction activity can be executed to develop the parcel for any activity, including agriculture. Land reclamation is defined to be a less expensive activity than the production of goods, transportation services, or labor. This ensures that reclamation is the default activity if developed land is taken out of production. There is no provision in the model for abandonment.

Summary of the Linear Programming Tableau

The model is premised on a set of embedded, annular zones surrounding a central import/export node. The advantages of this contrived, aggregate space model are the principal advantages provided by any monocentric urban model: computational efficiency and locational scarcity (Gordon and Moore, 1989). The mathematical specification of the model follows.

\[ X_{r,p}(i) = \text{the production of commodity } r \text{ in zone } (i) \text{ during period } p \text{ (endogenous);} \]
\[ X_{C,r,p}(i) = \text{the construction of facilities for the production of commodity } r \text{ in zone } (i) \text{ during period } p \text{ (endogenous);} \]
\[ X_{D,r,p}(i) = \text{the demolition of facilities for the production of commodity } r \text{ in zone } (i) \text{ during period } p \text{ (endogenous);} \]
\[ X_{T,p}(i) = \text{the level of transport capacity available in zone } (i) \text{ during period } p \text{ (endogenous);} \]
\[ X_{C,T,p}(i) = \text{the construction of transportation facilities in zone } (i) \text{ during period } p \text{ (endogenous);} \]
\[ X_{D,T,p}(i) = \text{the demolition of transportation facilities in zone } (i) \text{ during period } p \text{ (endogenous);} \]
\[ F_{r,p}^h(i) = \text{the flow of commodity } r \text{ outbound across boundary } h \text{ of zone } (i) \text{ during period } p, \text{ with } h = 1 \text{ denoting flows across the outer boundary of zone } (i) \text{ and } h = 2 \text{ denoting flows across the inner boundary of zone } (i) \text{ (endogenous);} \]
\[ X_{r,p}^-(i) = \text{the quantity of commodity } r \text{ exported out of the urban system from an export node located in zone } (i) \text{ during period } p \text{ (exogenous);} \]
\[ A(i) = \text{the area of zone } (i) \text{ (exogenous);} \]
\[ a_{q,r} = \text{the quantity of production input } q \text{ required per unit of commodity } r \text{ produced, with } q = 1 \text{ denoting land, } q = 2 \text{ denoting capital, } q = 3 \text{ denoting labor, and } q > 3 \text{ denoting intermediate inputs (exogenous);} \]
\[ a_{q,T} = \text{the quantity of production input } q \text{ required to produce transportation services capable of accommodating one unit of transport demand, with } q = 1 \text{ denoting land and } q = 2 \text{ denoting capital (exogenous);} \]
$c_{q,r} =$ the quantity of production input $q$ required to construct facilities sufficient to produce one unit of commodity $r$ (exogenous, $c_{1,r} = a_{1,r};$

$d_{q,r} =$ the quantity of production input $q$ required to demolish facilities sufficient to produce one unit of commodity $r$ (exogenous, $d_{1,r} = a_{1,r};$

$c_{q,T} =$ the quantity of production input $q$ required to construct facilities sufficient to produce one unit of transportation service (exogenous, $c_{1,T} = a_{1,T};$

$d_{q,T} =$ the quantity of production input $q$ required to demolish facilities sufficient to produce one unit of transportation service (exogenous, $d_{1,T} = a_{1,T};$

$g_r =$ the level of transportation system demand imposed by the transport of one unit of commodity $r$ across any zone boundary (exogenous);

$u =$ the rent on one unit of capital, i.e., the discount rate (exogenous);

$w =$ labor’s reservation wage, i.e., the wage available in the agricultural sector; and

$y =$ the rent on one unit of agricultural land (exogenous).

**Cost-minimizing Objective Function.** Select production activities and levels by zone over time to minimize the discounted sum of all production, transportation, residential, demolition, and construction costs,

\[
Z_{\text{primal}} = \sum_{p=1}^{P} (1 + u)^{-p} \cdot \left( \sum_{t} \left[ \sum_{r} (y \cdot a_{1,r} + u \cdot a_{2,r} + w \cdot a_{3,r}) \cdot X_{r,p}(i) \right] + (y \cdot a_{1,T} + u \cdot a_{2,T}) \cdot X_{r,T}(i) \
+ \sum_{r} (y \cdot a_{1,r} + u \cdot c_{2,r} + w \cdot c_{3,r}) \cdot X_{C,T,r}(i) \right) \\+ (y \cdot a_{1,T} + u \cdot d_{2,r} + w \cdot d_{3,r}) \cdot X_{D,T,r}(i) \right) \right) \right) 
+ u^{-1} \cdot (1 + u)^{-p} \cdot \left( \sum_{t} \left[ \sum_{r} (y \cdot a_{1,r} + u \cdot a_{2,r} + w \cdot a_{3,r}) \cdot X_{r,p}(i) \right] + (y \cdot a_{1,T} + u \cdot a_{2,T}) \cdot X_{T,p}(i) \right) \right) \right)
\]

Period-specific costs are assumed to be discounted at a rate matching the exogenous rent on capital. This means that the relative importance of profits accruing progressively farther into the future are progressively reduced. In the general case, the discount rate effectively defines the relevant time horizon. In this case, the time horizon has been truncated subject to a change in the coefficients for the final period. All export flows and costs are assumed to be fixed at levels defined for the final time period. Consequently, the costs specific to the final time period
represent infinite streams. These are treated as annuities, and each cost term is divided by the rent on capital. The cost terms associated with construction and demolition are logical exceptions to this rule, since these activities obviously cannot be replicated an infinite number of times. A better approach, however, is to repeat the economic and technical conditions associated with the end of the time horizon in the last two time periods. This ensures that the outputs for the final period will not include construction or demolition, but will define a simple stationary tail (Manne, 1970).

**Equilibrium Flow Constraints.** In any time period and zone, and for any commodity; the sum accounting for the commodity outflows to adjacent zones, exported out of the urban system, consumed as a production input by other activities, consumed as a transportation input, consumed as a construction input, and consumed as a demolition input must be equal to the sum accounting for the commodity inflows from adjacent zones and the quantity of the commodity produced in the zone.

In the special case of the center zone (the export node)

\[
F_{r, p}^1(1) + X_{r, p}^1(1) + \sum_q a_{r, q} \cdot X_{q, p}(1) + a_{r, r} \cdot X_{r, r}(1) + \sum_q c_{r, q} \cdot X_{C, r, p}(1) + c_{r, r} \cdot X_{C, r, r}(1) + \sum_q d_{r, q} \cdot X_{D, r, p}(1) + d_{r, r} \cdot X_{D, r, r}(1) \leq F_{r, p}^2(2) + X_{r, p}(1) \quad \text{(for all commodities } r \text{ and periods } p)\]

Minimum export constraints have been defined as inequalities rather than equalities in the interests of conservatism. Excess production may be less expensive than reconfiguring land uses. In a more general, profit-maximizing formulation, exogenous export levels might be replaced by exogenous prices (Moore and Wiggins, 1988).

In the special case of the outermost zone (the urban periphery)

\[
F_{r, p}^2(N) + \sum_q a_{r, q} \cdot X_{q, p}(N) + a_{r, r} \cdot X_{r, r}(N) + \sum_q c_{r, q} \cdot X_{C, r, p}(N) + c_{r, r} \cdot X_{C, r, r}(N) + \sum_q d_{r, q} \cdot X_{D, r, p}(N) + d_{r, r} \cdot X_{D, r, r}(N) = F_{r, p}^1(N - 1) + X_{r, p}(N) \quad \text{(for all commodities } r \text{ and periods } p)\]

In the general case of an intermediate zone

\[
\sum_k F_{r, p}^k(i) + \sum_q a_{r, q} \cdot X_{q, p}(i) + a_{r, r} \cdot X_{r, r}(i) + \sum_q c_{r, q} \cdot X_{C, r, p}(i) + c_{r, r} \cdot X_{C, r, r}(i) + \sum_q d_{r, q} \cdot X_{D, r, p}(i) + d_{r, r} \cdot X_{D, r, r}(i) = F_{r, p}^1(i - 1) + F_{r, p}^2(i + 1) + X_{r, p}(i) \quad \text{[for all commodities } r, \text{ periods } p, \text{ and zones } (i = 2, \ldots, N - 1)]\]

**Land-availability Constraints.** In any time period and zone, the land dedicated to production (including residential use), construction of production facilities,
demolition of production facilities, transportation facilities, construction of transportation facilities, and demolition of transportation facilities must be less than or equal to the buildable land available in the zone.

\[ \sum_{r} [a_{i,r} \cdot (X_{r,p}(i) + X_{C,r,p}(i) + X_{D,r,p}(i))] + a_{i,T} \cdot (X_{T,p}(i) + X_{C,T,p}(i) + X_{D,T,p}(i)) \] \leq A(i) 

[for all periods p and zones (i)]

Transportation System Supply and Demand Constraints. In any time period and zone, the supply of transportation service must be greater than or equal to the demand for service imposed by the outflow of goods and labor to adjacent zones and the inflow of goods and labor from adjacent zones.

In the special case of the center zone (the export node)

\[ X_{r,p}(1) \geq \sum_{r} 2^{-1} \cdot g_{r} \cdot (F_{r,p}^{1}(1) + X_{r,p}^{-}(1) + F_{r,p}^{2}(2)) \]  
(for all periods p)

In the special case of the outermost zone (the urban periphery)

\[ X_{r,p}(N) \geq \sum_{r} 2^{-1} \cdot g_{r} \cdot (F_{r,p}^{1}(N) + F_{r,p}^{1}(N-1)) \]  
(for all periods p)

In the general case of an intermediate zone

\[ X_{r,p}(i) \geq \sum_{r} 2^{-1} \cdot g_{r} \cdot \left( \sum_{h} F_{r,p}^{h}(i) + F_{r,p}^{1}(i-1) + F_{r,p}^{2}(i+1) \right) \]  
[for all periods p and zones (i = 2, ... , N - 1)]

Transportation costs include no congestion component, though congestion has been treated in the static case (Mills, 1972, 1974a, 1974b, 1975; Kim, 1983, 1986; Rho and Kim, 1989).

Interperiod Land-use Change Constraints. In any given zone and for any commodity; the sum in period p accounting for the land dedicated to the production of the commodity and the construction of its production facilities must equal the sum in period p + 1 accounting for the land dedicated to the production of the commodity and the demolition of its production facilities. The time needed to reconfigure land uses defines the natural period of the model.

\[ X_{r,p}(i) + X_{C,r,p}(i) = X_{r,p+1}(i) + X_{D,r+1}(i) \]  
[for all commodities r; periods p = 1, ..., P - 1; and zones (i)]

Nonnegativity Constraints. The endogenous variables describing production levels by period, zone, and commodity; transportation supply by period and zone; and interzonal flows by period, zone of origin, direction, and commodity must all be nonnegative.

\[ X_{r,p}(i), X_{C,r,p}(i), X_{D,r,p}(i), X_{T,p}(i), X_{C,T,p}(i), X_{D,T,p}(i), F_{r,p}^{h}(i) \geq 0 \]  
(for all commodities r, periods p, zones i, and both directions h)
The Structure of Land-use Change: Operation of the Interperiod Constraints

At any point in time, and within any given zone, the developed and undeveloped land areas plus the parcels subject to construction and demolition are necessarily constrained by the total area available in the zone. Any land use that persists at a fixed capital intensity from one time period to the next has no associated reclamation expense. Land needs to be reclaimed only in the event of temporal shifts between production activities.

Consider a simplified model that includes a single production activity characterized by a single technology. The general form of the interperiod land use constraints requires that, within any zone, the land area dedicated to this production activity during time period $p$ plus the land area being reclaimed from this activity via demolition during period $p$ must be equal to the land area dedicated to the production activity in time period $p - 1$ plus construction of new facilities in time period $p - 1$. Optimality dictates that there be no construction in time period $p - 1$ if the same type of facility is demolished in the same location during time period $p$, and that there would be no demolition in time period $p - 1$ if the same type of facility is constructed in the same location during time period $p$.

If it is optimal to decrease production in the zone during period $p$; then as noted above, the interperiod constraint ensures that the land taken out of production is cleared. If land is reclaimed in the zone during period $p - 1$, but it is not optimal to increase production in period $p$, then the land reclaimed during period $p - 1$ will remain undeveloped in period $p$.

This last possibility appears to contradict a fundamental finding by Fujita (1976) that, given replaceable capital and fixed demolition costs, vacant land is never created in an optimal dynamic trajectory. Costs are deferred by undertaking demolition only when construction is imminent. This (apparent) contradiction reflects a difference in the activities represented by the two models. Fujita's infinite horizon, continuous-time formulation is structurally similar to the dynamic Mills heritage model in several ways; but his separate representation of buildings and activities is too expensive to replicate in a numerical context. However, the discrete time model captures technological constraints on the rate at which land use change can occur that are not conveniently treated in Fujita's continuous time formulation.

The Dual Problem

The objective function in Equation (10), combined with the constraints in Equations (11) through (19), form a multiperiod linear program that is decomposable across time periods. Minimizing the objective function identifies an efficient assignment of activities to zones and time periods, including transportation facilities for commodity and labor flows. The dual objective function maximizes the value of urban output minus the value of developed land (Hartwick and Hartwick, 1974, 1975),

$$Z_{\text{dual}} = \sum_p \left[ \sum_i X^*_{r,p}(1) \cdot v^*_{r,p} + \sum_i A(i) \cdot v^*_{1,p}(i) \right]$$
Dual variable \( v_{r,p}^- \) is nonnegative, and is the value of resources used to meet the last unit of final demand for good \( r \) in period \( p \). Dual variable \( v_{1,0}^- \) is nonpositive, and is the endogenous component of the unit land value in zone \( (i) \).

In the static problem, the dual constraints require that profits from all urban activities be nonpositive. The extension to the multiperiod case is straightforward. Subtracting the left-hand side of the equilibrium flow constraints from the right, and the right-hand sides of the transportation and land-use change constraints from the left, the general form of the dual constraint for variable \( X_{r,p}(i) \) is

\[
-(s_{r,p-1}(i) + v_{r,p}(i)) - \sum_k d_{k,r} \cdot v_{k,p}(i) + s_{r,p}(i) 
\leq (1 + u)^{-p} \cdot (y \cdot a_{1,r} + u \cdot a_{2,r} + w \cdot a_{3,r})
\]

[for all commodities \( r \); services \( C, D, \) and \( T \); periods \( p = (2, \ldots, P-1) \); and zones \( (i) \)]

The dual variable \( v_{r,p}(i) \) is nonpositive, and is the equilibrium price for commodity \( r \) in zone \( (i) \), period \( p \). Dual variable \(-s_{r,p-1}(i)\) is unrestricted in sign, and is the value of reconfiguring land uses between period \( p - 1 \) and period \( p \) to permit production of commodity \( r \) in zone \( (i) \). And dual variable \( s_{r,p}(i) \) is the value of reconfiguring land uses between periods \( p \) and \( p + 1 \) given production of commodity \( r \) in zone \( (i) \).

Expression (21) holds as an equality when the equilibrium production of commodity \( r \) in zone \( (i) \) is greater than zero. If commodity \( r \) is not produced in zone \( (i) \), then there is slack in the dual constraint, indicating the value of the resources that would be needed to make production possible exceeds the value of the output that would be produced. If the sum \(-s_{r,p-1}(i) + s_{r,p}(i)\) is positive, the equilibrium price for commodity \( r \) in zone \( (i) \) could be that much less relative to the static case, and producing commodity \( r \) in zone \( (i) \) would remain efficient. If the sum \(-s_{r,p-1}(i) + s_{r,p}(i)\) is negative, the equilibrium price for commodity \( r \) in zone \( (i) \) would have to be that much greater relative to the static case before production would be efficient in this location. In either case, land uses in the previous and subsequent periods are simultaneously constraining the system’s production schedule. Corresponding interpretations extend to the dual conditions imposed on transportation facilities, and construction and demolition activities.

It is illuminating to examine the dual constraints associated with the primal transportation flow variables, particularly those defining reverse commutation. Consider \( F^1_{S,r}(i) \), an outward labor flow from zone \( (i) \) to adjacent zone \( (i + 1) \). The corresponding dual constraint is

\[ F^1_{S,r}(i) \]

More generally, the right-hand side of this dual constraint would consist of unit linehaul costs for commodity \( r \). However, because there are no congestion costs in this formulation, linehaul costs have been assumed to be uniform across all flows. Transportation costs not associated with inputs of land, capital, and materials are assumed to be captured by the magnitude of the coefficients \( g_r \). This assumption can be relaxed by defining coefficients \( f_r \), defined for the simplest problem, that represent the (exogenous) linehaul cost of moving 1 unit of commodity \( r \) across the boundary between adjacent zones. These coefficients would appear in the primal objective function as terms \( \Sigma_r (1 + u)^{-p} \Sigma_i \Sigma_h F^*(r) \).
(22) \[ v_{h,i}(i + 1) - v_{h,i}(i) - 2^{-1} \cdot g_3 \cdot v_{r,i}(i) - 2^{-1} \cdot g_3 \cdot v_{r,i}(i + 1) \leq 0 \]

(for periods \( p = (2, \ldots, P - 1) \); and zones \( i \))

That is,

(23) \[ \text{Wage}(i + 1) - \text{Wage}(i) \leq \text{Transport Labor Cost}(i, i + 1) \]

It follows from complementary slackness that if (23) holds as an inequality, then reverse commutation must be zero. If (23) holds as an equality, then reverse commutation is possible. If \( \text{Wage}(i + 1) \) is always less than \( \text{Wage}(i) \), then the inequality holds since the right hand side is always positive. This provides a fundamental theorem of reverse commutation. Reverse commutes cannot be associated with a wage gradient that decreases from the center of the city outward. More strongly, if reverse commutation is to occur, it is necessary that wages increase with distance from the center. Further, the wage gradient must be sufficiently positively sloped. This is reasonable. If a worker living in zone \( i \) is to be induced to work in zone \( i + 1 \), the wage in zone \( i + 1 \) must be sufficient to compensate the worker for his forgone wage in zone \( i \) plus the cost of commuting to zone \( i + 1 \). Positively sloped wage gradients are the shadow of reverse commutation.

3. MULTIPERIOD RESULTS

A five-zone five-period Mills heritage model defines a constraint-intensive linear program with 430 variables, 210 constraints, and 1,440 nonzero coefficients in the constraint set; resulting in a tableau density of approximately 1.9 percent. Five time periods were specified even though only three are necessary to completely execute land use changes to better treat temporal boundary effects. The results associated with the first time period are contrived. Land uses in this period are not constrained by an existing built environment. Similarly, the logic of construction and demolition activities makes it necessary to replicate (undiscounted) parameter values across the last two time periods. Construction and/or demolition may be ongoing in the second to last period. The new facilities or vacancies that result can meaningfully be treated as perpetual outcomes, but demolition and construction activities cannot. Thus, the problem needs to be structured with a final, redundant time period. Further, specifying multiple periods and zones is a conservative approach. The formulation provides many degrees of freedom for the optimal solution, and maximizes the likelihood that the optimal solution will not include reverse commutation.

Various versions of this linear program were solved using the LINDO 85 optimization package implemented on the University of Southern California’s IBM 3090 mainframe computer. Optimization typically required a few minutes of CPU time. This computational intensity is not surprising, because the dynamic model is a staircase linear program that optimizes a set of technological options linked over both time and space.

Figure 4 compares the optimal stocks for each time period. Because the final demand vectors for periods 4 and 5 are identical, the fifth period necessarily replicates the pattern of the fourth absent construction and demolition. The results
are conceptually appealing. Capital-intensive production activities are prevalent at the urban interior. Land-intensive activities prevail at the urban periphery. A relatively small amount of land is dedicated to residential use, because housing is assumed to be the least land-intensive activity. In period 1, there is vacant land in zones 3, 4, and 5. As noted above, the results for period 1 are contrived. They cannot be interpreted in the same light as results associated with subsequent time periods.

Over time, land uses at the urban interior are reconfigured to accommodate
the increase in land-intensive exports. Most of the land use change occurs in zone 3. During period 2, some of the production facilities for good 2 are demolished while new facilities for good 1 are constructed on previously vacant land. Transportation facilities in zones 1 and 2 are demolished to make land available for new uses, one of which, in the case of the center zone, is housing. This shift to housing occurs despite vacant land at the urban periphery.

Figure 5 compares the optimal transportation flows each period. In periods 1

**FIGURE 5:** Optimal Transportation Flows by Origin, Destination, and Period.
and 2, all labor flows are directed toward the city center. Periods 4 and 5 provide the counternintuitive results put forth in Section 1. These include both reverse commutation; and, in periods 4 and 5, a reverse flow of the capital-intensive export commodity.

4. CONCLUSIONS AND EXTENSIONS

These general equilibrium results model the spatial and temporal operation of a hypothetical, multisector urban economy. Location, production, transportation, and infrastructure decisions are optimized jointly over time in response to exogenous shifts in export prices. The dynamic Mills heritage model is a pure market model, the optimal capital replacement and production strategies obtained for the dynamic Mills heritage model include land-use configurations that appear to be suboptimal if viewed from a static perspective (Moore and Wiggins, 1990). If the market price of capital is assumed to define the social discount rate, then no public policy prescriptions are implied by the model outputs. The possibility of market failure is precluded by the nature of the formulation.

A Composite Export Good

Though the numerical example exploits the existence of alternative export goods, the dual constraints associated with reverse commuting flows do not imply that this is a necessary condition for the existence of reverse commutation. All that is required is that the cost of providing labor in a given zone exceeds the cost of importing labor from adjacent zones. This condition could be met with a composite export good. The temporal pattern of demand is much more important than the number of commodities represented in the model.

Nonequivalence of Static and Long-run Perspectives

The assumption of an infinite time horizon does not imply that land uses in the final time period will replicate a static equilibrium. So long as the interest rate is strictly positive, an infinite stream of uniform net revenues will have a finite present value, and the objective-function coefficients for the final period will be finite. As the value of the interest rate approaches zero, the parameters of the final time period increase quickly. Net revenues accruing in the final period will begin to dominate the cost of land-use change, and the outputs of the final time period will approach those of the corresponding static formulation. This leads to the conclusion that static and long-run results are equivalent only if the economic and technical conditions of the system never have been nor ever will be subject to change. The conventional static perspective, i.e., the absence of future changes, is a necessary but not sufficient condition. It is precisely this restrictive assumption that prescribes forward commutation as an optimal labor flow. The prescription fails when optimal land-use change is modeled explicitly.

Related Empirical Research

Real cities are considerably more complicated than the abstract representations provided by urban economic models. In particular, cities are obviously not monocentric. Much of the literature recognizes this fact, though the standard approach to acentric systems is to define subcenters around which familiar negative rent, wage, and density gradients should be observable. Under these
assumptions, neither reverse commutes nor commuting cross hauls would take place; but they do. Real worktrip origin-destination matrices have very few empty cells.

A number of empirical tests have been suggested to validate the assumptions of urban economic models, and most of these focus directly on the role of commutation. In a static, monocentric model, the sole reason for expecting a negative rent gradient is the need to compensate distant households for higher commuting costs (Muth, 1969). Consequently, many hedonic models of house prices that include property characteristics, group memberships, neighborhood attributes, and other descriptors have been estimated in the search for a house price or rental gradient that is negatively sloped with respect to the central business district.

A shorter line of research explores the micro-level trade-off between housing expenditures and commuting costs. Recent work by Richardson, Gordon, and Choi (1991) uses commuting information from the American Housing Survey to test for a statistically significant trade-off between housing expenditures and commuting costs for single worker rental households. Though they detected a weak relationship for the poorest households residing in the densest cities, their conclusion is that rent differentials cannot usually be explained by commuting cost savings.

A third line of inquiry relies on the concept of wasteful commuting. In an influential paper, Hamilton (1982) applied an optimization model to derive the minimum average commute obtained by reassigning workers to either different houses or different jobs. Based on data for several U.S. cities, he concluded that 87 percent of observed vehicle commuting miles were wasteful. White (1988) extended Hamilton’s model to allow for subcentering and nonubiquitous transportation. The extension reduced the estimate of wasteful commuting to 11 percent, a figure Hamilton contests (1989). More recent research by Giuliano, Small, and Jun (1991) disaggregates space into many more work and residential zones, and imposes occupational constraints on job matches. These occupational considerations increased the minimum required commutes computed for Los Angeles by only four percent, leaving 50 percent or more of observed commuting distances unexplained. Data constraints, however, limit the number of occupational categories to seven, and the question of whether or not a highly disaggregate representation of employment would substantially increase minimum feasible commutes remains open.

These ongoing research efforts demonstrate substantial investigative innovation, and considerable scholarly interest in the subject of efficient commutation and location. The literature, however, rarely examines the importance of the static perspective implicit in most urban economic models. It is not surprising that the bulk of this work often concludes with findings that either contradict or appear to be unrelated to the conventional wisdom. Most of the questions being investigated are moot. Commuting decisions are not explained by static models, monocentric or otherwise. Even in the highly contrived context of a monocentric model, explicit treatment of time and attendant changes identify efficient land-use and travel patterns that, while empirically plausible, will never be replicated by static formulations.
REFERENCES


## APPENDIX

### TABLE A1: Technical Coefficients

<table>
<thead>
<tr>
<th>$[a_{sk}]$ Inputs</th>
<th>Outputs $r = 3$</th>
<th>Outputs $r = 4$</th>
<th>Outputs $r = 5$</th>
<th>Outputs $T$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$q = 1$</td>
<td>0.100</td>
<td>1.600</td>
<td>0.600</td>
<td>0.200</td>
</tr>
<tr>
<td>$q = 2$</td>
<td>0.100</td>
<td>0.200</td>
<td>3.000</td>
<td>2.400</td>
</tr>
<tr>
<td>$q = 3$</td>
<td>0.050</td>
<td>0.275</td>
<td>0.275</td>
<td>0.000</td>
</tr>
<tr>
<td>$q = 4$</td>
<td>0.010</td>
<td>0.005</td>
<td>0.300</td>
<td>0.000</td>
</tr>
<tr>
<td>$q = 5$</td>
<td>0.100</td>
<td>0.175</td>
<td>0.000</td>
<td>0.000</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$[a_{sk}]$ Inputs</th>
<th>Outputs $C, r = 3$</th>
<th>Outputs $C, r = 4$</th>
<th>Outputs $C, r = 5$</th>
<th>Outputs $C, T$</th>
<th>Outputs $D, r = 3$</th>
<th>Outputs $D, r = 4$</th>
<th>Outputs $D, r = 5$</th>
<th>Outputs $D, T$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$q = 1$</td>
<td>0.100</td>
<td>1.600</td>
<td>0.600</td>
<td>0.200</td>
<td>1.000</td>
<td>1.600</td>
<td>0.600</td>
<td>0.200</td>
</tr>
<tr>
<td>$q = 2$</td>
<td>0.200</td>
<td>0.400</td>
<td>6.000</td>
<td>4.800</td>
<td>0.500</td>
<td>1.000</td>
<td>15.000</td>
<td>12.000</td>
</tr>
<tr>
<td>$q = 3$</td>
<td>0.060</td>
<td>0.300</td>
<td>0.300</td>
<td>0.000</td>
<td>0.050</td>
<td>0.200</td>
<td>0.200</td>
<td>0.000</td>
</tr>
<tr>
<td>$q = 4$</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>$q = 5$</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
</tr>
</tbody>
</table>

### Factor prices
- $w = 1.0$
- $y = 0.8$
- $u = 1.2$

### Transportation weights
- $g_3 = 0.1$
- $g_4 = 0.5$
- $g_5 = 0.2$

### Zone (ring) areas
- $A(1) = \pi$
- $A(2) = 3 \cdot \pi$
- $A(3) = 5 \cdot \pi$
- $A(4) = 7 \cdot \pi$
- $A(5) = 9 \cdot \pi$

### Export Schedule
<table>
<thead>
<tr>
<th>Period</th>
<th>$X_{tq}$</th>
<th>$X_{ts}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$p = 1$</td>
<td>20.0</td>
<td>20.0</td>
</tr>
<tr>
<td>$p = 2$</td>
<td>23.0</td>
<td>17.0</td>
</tr>
<tr>
<td>$p = 3$</td>
<td>25.0</td>
<td>15.0</td>
</tr>
<tr>
<td>$p = 4$</td>
<td>27.0</td>
<td>13.0</td>
</tr>
<tr>
<td>$p = 5$</td>
<td>27.0</td>
<td>13.0</td>
</tr>
</tbody>
</table>

These coefficients were derived from a single set of hypothetical inputs created and subsequently revised by E. Mills (1972, 1974a, 1974b, 1975), modified by Hartwick and Hartwick (1974, 1975), condensed by Kim (1978a, 1978b, 1979), and extrapolated by Moore and Wiggins (1988, 1990). These coefficients are not empirical. Their relative magnitudes are reasonable, but their values are, with minor exceptions, hypothetical. An empirical application of the Mills heritage approach has recently been completed by Rho and Kim (1989).