Acknowledgements

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Curvilinear Motion

Recall that we described a general motion along a curved path (curvilinear motion) through space through the position vector \( \vec{r}(t) \).

In terms of a reference frame with an associated Cartesian coordinate system\(^2\), we have

\[
\vec{r}(t) = x(t)\hat{i} + y(t)\hat{j} + z(t)\hat{k}
\]

As compared to straight-line motion, we now have up to three scalar quantities to consider (2 for 2D).

A key feature is that (like equilibrium) the motion along one axis is independent of the motion along the other axes.

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\(^2\) In Section 13.7, the book covers polar and cylindrical coordinates. We will not cover this.
Velocity and acceleration are again obtained by differentiating, where differentiating a vector simply means differentiating each scalar component:

\[
\vec{v}(t) = \frac{d\vec{r}(t)}{dt} = \frac{dx(t)}{dt} \hat{i} + \frac{dy(t)}{dt} \hat{j} + \frac{dz(t)}{dt} \hat{k}
\]

We denote the velocity components by

\[
\begin{align*}
v_x &= \frac{dx}{dt} \\
v_y &= \frac{dy}{dt} \\
v_z &= \frac{dz}{dt}
\end{align*}
\]

Then,

\[
\vec{v}(t) = v_x(t)\hat{i} + v_y(t)\hat{j} + v_z(t)\hat{k}
\]

Similarly,

\[
\vec{a}(t) = \frac{d\vec{v}(t)}{dt} = \frac{dv_x(t)}{dt} \hat{i} + \frac{dv_y(t)}{dt} \hat{j} + \frac{dv_z(t)}{dt} \hat{k}
\]

\[
= a_x(t)\hat{i} + a_y(t)\hat{j} + a_z(t)\hat{k}
\]
The acceleration components are

\[ a_x = \frac{dv_x}{dt} \]

\[ a_y = \frac{dv_y}{dt} \]

\[ a_z = \frac{dv_z}{dt} \]

We will only look at 2D problems, so we only worry about two components (x and y) of position, velocity, and acceleration.

**Example:** Problem 13.75 from the textbook.

To drop an object onto a target from a height of \( h = 30 \text{ m} \) with a velocity of \( v_0 = 40 \text{ m/s} \), what is the distance \( d \) at which the object should be dropped?

Strategy: We need to know how long it will take for the object to fall (y-direction) and then, in that time, how far it will move horizontally (x-direction). In the absence of other information, we will neglect drag and assume acceleration in y-direction is given by acceleration due to gravity.

Select reference frame as usual (x-direction horizontal and y-direction vertical), with the origin on the ground and \( x = 0 \) corresponding to the position of the plane at \( t = 0 \).
First, consider the $y$-component of the motion:

$$a_y = -g$$

$$v_y(t) = \int a_y \, dt$$
$$= -gt + C_1$$

$$v_y(0) = 0 \Rightarrow C_1 = 0$$

$$y(t) = \int v_y \, dt$$
$$= \int (-gt) \, dt$$
$$= -\frac{1}{2} gt^2 + C_2$$

$$y(0) = h \Rightarrow C_2 = h = 30 \text{ m}$$

Determine drop time using $y(t_{drop}) = 0$:

$$y(t_{drop}) = -\frac{1}{2} g t_{drop}^2 + h = 0$$

$$t_{drop} = \sqrt{\frac{2h}{g}}$$

$$= \sqrt{\frac{2(30 \text{ m})}{9.81 \text{ m/s}^2}}$$

$$= 2.473 \text{ s}$$
Now, consider the \( x \)-component of the motion:

\[
x(t) = v_0 t
\]

\[
x(t_{\text{drop}}) = v_0 t_{\text{drop}}
\]

\[
= \left( 40 \ \text{m/s} \right) (2.473 \ \text{s})
\]

\[
= 98.9 \ \text{m}
\]

\[
d = 98.9 \ \text{m}
\]