Acknowledgements

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Introduction to Mechanics

Mechanics is essentially the study of forces and their effects. It forms the basis of all modern engineering, which rests on mathematical modeling.

Statics is the study of objects at rest. Dynamics is the study of objects in motion. Newton’s laws form the basis for these analyses.

We can use mechanics to predict forces in structures (statics), the trajectory of objects\(^2\) (dynamics), and much more.

You can build on the concepts from this course to study fluid flow, deformation of solids, and so much more…

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2 Sketch orbit and refer to early works by Greek Philosophers (400 B.C. to A.D. 500), Tycho Brahe, Johannes Kepler, and Galileo Galilei (late 1500’s to early 1600’s).
Problem Solving

General steps:

- Identify given information and what you must determine.
- Develop a strategy: identify appropriate principles and equations.
- Predict the answer.
- Solve the equations, interpret your results, and compare with your prediction.

Hone your problem solving skills with practice!

Five basic tools in this course:

1. Newton’s laws
2. Vector algebra (dot product, cross product, etc.)
3. Basic geometry and trigonometry
4. Free body diagrams
5. Simple differential and integral calculus.

Numbers and Units

“Significant digits” refers to the number of meaningful digits. This is typically determined by the accuracy of a measurement. In the textbook, data and answers are almost always expressed to 3 significant digits. You must do the same!

Use higher precision for intermediate steps to avoid round-off errors.
Examples:

- 21.6 km/h
- 62,700 lb/ft²
- \( \alpha = 0.175 \) rad

<table>
<thead>
<tr>
<th></th>
<th>International System (SI)</th>
<th>U.S. Customary Units</th>
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</thead>
<tbody>
<tr>
<td>distance</td>
<td>meters (m)</td>
<td>feet (ft)</td>
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<tr>
<td>time</td>
<td>seconds (s)</td>
<td>&quot;&quot;</td>
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<tr>
<td>mass</td>
<td>kilograms (kg)</td>
<td>slugs</td>
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<tr>
<td>angle</td>
<td>radians (rad) or degrees (deg)</td>
<td>&quot;&quot;</td>
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<tr>
<td>force</td>
<td>Newtons (N)</td>
<td>pounds (lb)</td>
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<tr>
<td>energy</td>
<td>newton-meters (N·m)</td>
<td>foot-pounds (ft·lb)</td>
</tr>
</tbody>
</table>

Note that distance, time, and mass are fundamental.

In this course, we will mainly use SI units³.

Example⁴

What are Newtons in fundamental units?

\[ 1 \text{ N} = (1 \text{ kg})(1 \text{ m/s}^2) = 1 \text{ kg} \cdot \text{m/s}^2 \]

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³ U.S. Customary Units should be banned!
⁴ Pose the question to students. Nice segue to Newton’s 2nd Law, which is covered next.
Converting and Determining Units

Converting units is straightforward but must be done with care.

Example 5 (converting units)

\[
1 \text{ mi/h} = \left(1 \frac{\text{mi}}{\text{h}}\right) \left(\frac{5280 \text{ ft}}{1 \text{ mi}}\right) \left(\frac{1 \text{ h}}{3600 \text{ s}}\right) = 1.47 \frac{\text{ft}}{\text{s}}
\]

Example 6 (determining units)

Given:

\[
v = \sqrt{\frac{gR^2}{r}}
\]

where the units of \(g\), \(R\), and \(r\) are \([g] = \frac{\text{m}}{\text{s}^2}\), \([R] = \text{m}\), and \([r] = \text{m}\)

Find:

(a) the units of \(v\)

(b) the value of \(v\) if \(R = 6370\ \text{km}\), \(r = 6670\ \text{km}\), and \(g = 9.81\ \text{m/s}^2\)

(c) to what physical problems does this pertain?

5 Emphasize that this is unit conversion, so the rules for significant figures do not apply (otherwise 1 mi/h = 1 ft/s).

6 Active learning example.
Solution:

(a) \[ v = \sqrt{\frac{[g][R]^2}{[r]}} = \sqrt{\frac{m}{s^2} \left( \frac{m}{g} \right)^2} = \sqrt{\frac{m^2}{s^2}} = \frac{m}{s} \]

(b) \[ v = \sqrt{\frac{9.81 \frac{m}{s} (6370 \text{ km})^2 \left( \frac{1000 \text{ m}}{1 \text{ km}} \right)^2}{(6670 \text{ km}) \left( \frac{1000 \text{ m}}{1 \text{ km}} \right)^2}} = \frac{725.22 \text{ m}}{s} = 7.73 \times 10^3 \frac{\text{m}}{s} \text{ or } 7.73 \frac{\text{km}}{s} \]

(c) Celestial body (orbiting satellite)

Angles

Watch out for angles!

Examples:

\[ 45 \text{ deg} = 45 \text{ deg} \left( \frac{\pi \text{ rad}}{180 \text{ deg}} \right) = 0.785 \text{ rad} \]

\[ x = \frac{\sin \theta}{l} \approx \frac{\theta}{l} \text{ for small } \theta \]

\( \theta \) must be in radians!
Newton’s Laws

Newton’s laws form the basis for Newtonian mechanics.

1st Law: An object will remain at rest or in uniform motion in a straight line unless acted upon by an external force.

\[ \vec{F} = \frac{d}{dt}(m \vec{v}) \]

If \( \frac{dm}{dt} = 0 \) then \( \vec{F} = m \vec{a} \).

3rd Law: All forces in the universe occur in equal but oppositely directed pairs.

Newtonian Gravitation

The gravitational force \( F \) between two particles of mass \( m_1 \) and \( m_2 \) that are separated by a distance \( r \) is

\[ F = \frac{G m_1 m_2}{r^2} \]

\( G = 6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2 \) is called the universal gravitational constant.

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7 These are listed in §1.1 of the textbook and do not necessarily need to be written out. These statements of the first and third laws are similar to the statements found here: http://hyperphysics.phy-astr.gsu.edu/hbase/Newt.html.
8 Compare with Einstein and Schrödinger.
9 First Law is special case of the Second Law, one for which the net external force is zero.
10 Nice segue to vectors! In words, the sum of the forces on a particle is equal to the rate of change of the linear momentum of the particle. If the mass is constant, the sum of the forces is equal to the product of the mass of the particle and its acceleration.
We can use this equation to approximate the weight $W$ of an object at sea level.

$$W = \frac{Gmm_E}{r_E^2}$$

Upon defining $g = \frac{Gm_E}{r_E^2}$, one finds $W = mg$.

Acceleration due to gravity varies on Earth’s surface, but we typically assume\(^\text{11}\) $g = 9.81 \text{ m/s}^2 = 32.2 \text{ ft/s}^2$.

\(^\text{11}\) Ask students if they recall these numbers. Good example of significant digits.