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A wavelet-based time-varying adaptive LQR algorithm for structural control

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Abstract

A modified form of the conventional linear quadratic regulator (LQR) control algorithm has been proposed in this paper. The formulation of the modified LQR algorithm uses the information derived from the wavelet analysis of the response in real time, to obtain the local energy distribution over frequency bands. This information reflecting the effect of excitation on the structural system are used to adaptively design the controller by updating the weighting matrices to be applied to the response energy and the control effort. The optimal LQR control problem is solved for each time interval with updated weighting matrices, through the Ricatti equation, leading to time-varying gain matrices. The advantage of the proposed control algorithm is that, it does not require an a priori (offline) choice of the weights as in the classical case and adaptively calculates the gains using the weights decided on the response characteristics in real time (online). The proposed wavelet-based adaptive time-varying LQR (TVLQR) controller is applied to both single-degree-of-freedom (SDOF) and multi-degree-of-freedom (MDOF) systems and the results are compared with the case of conventional LQR controller. Simulations based on single and multiple controllers indicate that the proposed TVLQR controller achieves significant reduction in the displacement response of the structures as compared to the reduction obtained from the use of LQR controller in some cases with acceptable increment in peak control force and marginal increment in control energy demand.

Keywords: LQR; Adaptive structural control; Wavelet analysis; Time-varying weights; Time-varying gains

1. Introduction

A considerable amount of research has been carried out over the past few decades for structural control applications [20,14,15,8–10] including optimal control theory. However, algorithms based on classical control algorithms such as linear quadratic regulator (LQR) suffer from some inherent shortcomings for structural applications, which have been acknowledged by researchers and improvement on the classical algorithms has been proposed [17,13,19,1].

One of the major shortcomings of the LQR algorithm for application to forced vibration control of structures is its inability to explicitly account for the excitation. While this is not of much concern for free the vibration scenarios or for stabilizing structural systems to de-stabilizing forces, the earthquake or wind excited structures are subjected forced vibration and for the control, the effect of the external excitations needs to be accounted for in such cases. However, difficulty in solving the optimal control problem lies in numerically solving matrix equations backward in time, which requires the excitation to be known a priori. Since this is not available generally for a structural system subjected to unknown excitations the usual approach has been to solve the optimal control problem ignoring the excitation term to obtain the gains to be applied to get the desired control target.

Only a few studies so far have accounted for the effect of the excitation term in the optimal control formulation for structural systems. Wu et al. [17] included the forcing term from a single earthquake in their formulation. The formulation lacks generality as it may not ensure optimal behavior for other earthquakes. Wu and Nagarajaiah [18] formulated a predictor–corrector algorithm for the solution of the optimal control problem with applications to base isolated structures and structures with tuned mass damper (TMD). In these studies,

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the weighting matrices assigned to the cost of reduction of the response states and the control effort are chosen independent of the optimization process and the results may be specific to earthquakes. In order to account for the possible variability in different earthquakes and include the selection of the weighting matrices in the optimization process, Panariello et al. [13] had proposed a method based on the training on an ensemble of earthquake ground motions. Though, this method is capable of accounting for the forced excitations and optimize the weighting, it would still need a representative database and is essentially offline. Further, excitations such as earthquakes could be highly non-stationary, with time-variant energy and frequency content. The evaluation of the time-variant energy and frequency content of the structural responses, which indirectly reflect the effect of the external excitation is necessary and there is a necessity of updating the LQR weighting matrices by tracking the nature of the response.

This paper proposes a wavelet-based adaptive linear quadratic regulator (LQR) formulation for the optimal control problem. No information on the excitation is required apriori. Wavelet-based techniques have become popular for time-frequency analysis of vibration problems [12,2,16,7,3,11,4]. The use of multi-resolution wavelet analysis as a time-frequency tool helps to resolve the response time signals in different frequency bands. This provides the time-varying energy in different frequency bands. The weighting matrices are updated online by using a scalar multiplier or a factor according to the energy in the different frequency bands over a time window and the controller gains are optimized over each of these windows progressively in time. The weighting matrices are scaled by a factor according to the magnitude of the energy and whether it corresponds to a resonant or a non-resonant band of frequency, which indirectly reflects on the effect of the external excitation. Also, the chosen updated weighting matrices are more robust and are not affected by the uncertainties in the system parameters as they depend on a band of frequency. Since, the update of the gains takes place at an interval of every window width, the control has low-frequency switching requirements as compared to a continuous updated controller like classical LQR. The use of discrete wavelet transform (DWT) using the multi-resolution analysis (MRA) algorithm makes the proposed controller fast and eliminates the problem related to time delay.

The proposed wavelet-based adaptive time-varying LQR (TVLQR) control has been applied to single-degree-of-freedom (SDOF) and multi-degree-of-freedom (MDOF) systems to shows its effectiveness in reducing the seismic vibration response of structures.

2. System equations and quadratic cost functional of LQR

Consider a state-space representation of the equations of motions of an N-DOF linear structural system subjected to a base acceleration $\ddot{z}$, represented by the following set of matrix equations

$$\{\dot{x}\} = [A]\{x\} + [B]\{u\} + [E]\ddot{z} \tag{1}$$

and

$$\{u\} = -[G]\{x\}. \tag{2}$$

In Eqs. (1) and (2), the $2N$-dimensional state vector is expressed as

$$\{x\} = \begin{Bmatrix} y(t) \\ \dot{y}(t) \end{Bmatrix} \tag{3}$$

and $\{u\}$ is the control vector of dimension $M \times 1$. The system matrix $[A]$ is of order $2N \times 2N$, the control influence vector $[B]$ is of order $2N \times M$, the excitation influence vector $[E]$ of order $2N \times 1$ and the gain matrix $[G]$ is of the order of $2N \times M$. The state matrix, control and the excitation influence vectors are given by

$$[A] = \begin{bmatrix} 0 & I \\ -M^{-1}K & -M^{-1}C \end{bmatrix}; \tag{4}$$

$$[B] = \begin{bmatrix} 0 \\ M^{-1}D \end{bmatrix} \tag{5}$$

and

$$[E] = \begin{bmatrix} [0]_{N\times 1} \\ [I]_{N\times 1} \end{bmatrix} \tag{6}$$

respectively. In Eqs. (4) and (5), the $[M]$, $[K]$ and $[C]$ are the mass, stiffness and damping matrices respectively. To formulate an optimal control problem, an appropriate cost functional incorporating the two components (i) the states to be controlled and (ii) the control effort has to be constructed with weightings on the two parts. On substituting Eq. (2) in Eq. (1) and defining the functional over the duration $(0, t_f)$

$$J = \int_0^{t_f} [\{x\}^{\mathrm{T}}[Q]\{x\} + \{u\}^{\mathrm{T}}[R]\{u\}]\mathrm{d}t \tag{7}$$

to be minimized subject to the constraints given by Eqs. (1) and (2) leads to the classical LQR formulation. In Eq. (7), $[Q]$ and $[R]$ are the weighting matrices to be applied to the response and control energy respectively. For an infinite terminal time, the solution to this problem is given by the algebraic Ricatti equation from where the gain matrix $[G]$ can be obtained.

3. Wavelet-based adaptive TVLQR formulation

The weighting matrices $[Q]$ and $[R]$ in the classical LQR formulation represent the relative importance to be assigned to the structural response or the control effort respectively. A relative larger weight would impose a higher penalty on the corresponding term for optimization of the total cost functional. Hence, if the reduction of the structural response is of prime concern irrespective of the cost of control or even at the expense of higher cost of control, a lower weighting force is to assigned to term associated with the calculation of the control effort and vice versa.

This concept of using the weighting matrices works well in the classical LQR case when there is no necessity of updating the weighting matrices or changing the assignment of relative importance of the structural response and the control effort. However, there may be cases for systems subjected to external excitations such that the response has time-varying frequency content due to the variation in the excitation frequency over time. The closeness of the frequency of excitation to the natural frequencies of the system and its amplitude may require a revised weighting function on the term associated with the control to have a better effectiveness.

Earthquake excitations are known to be highly non-stationary in both amplitude and frequency content. The excitation causes the structural response to be non-stationary with the possibility of short duration impulsive nature to sudden change in the frequency to long periods. To account for this, the classical LQR formulation has to be modified to accommodate the importance of the local time-varying frequency content, which if close to the natural frequencies of vibration of the system results in a large response locally. These transient phenomena could be captured by the use of time-frequency analysis.

Wavelet analysis has been recognized as one of the most powerful and versatile time-frequency tools, which has the ability to resolve frequency locally in time. Apart from being able to provide the most optimized time-frequency resolution it has the advantage of providing a variety of different basis functions for analysis and fast exact reconstruction techniques using multi-resolution analysis. The results from the wavelet analysis of the response in real time has been used to modify the classical LQR problem. The use of wavelet analysis enables the capturing of the local variation in the frequency content with an adaptive updating of the weighting in the TVLQR formulation.

4. Modified cost functional

The total time duration of the response under consideration $(0, t_f)$ is sub-divided into windows, with the ith window being (t_{i-1}, t_i). For each of the windows, the cost performance integral is developed with weighting matrices $[Q]_i$ and $[R]_i$ as

$$J_i = \int_{t_{i-1}}^{t_i} [\{x\}^{\mathrm{T}}[Q]_i\{x\} + \{u\}^{\mathrm{T}}[R]_i\{u\}]\mathrm{d}t \tag{8}$$

which is to be minimized subject to the constraints given by Eq. (1) and the control equation

$$\{u\} = -[G]_i\{x\}. \tag{9}$$

The matrix $[G]_i$ is the gain to be applied for the ith window to achieve the desired control of the response. The gain matrix can be obtained for the windowed interval as a solution of the Ricatti matrix differential equation, which in a steady-state case leads to the solution of Ricatti algebraic equation for the classical LQR problem with the weighting matrices $[Q]_i$ and $[R]_i$. These matrices are to be updated for each window resulting in the gain matrix $[G]_i$ for each window.

In the modified control problem, a windowed approach is followed where the total duration of the response is sub-divided into a number of time windows and the cost functional is minimized over each of these windows. Since the cost functional has been modified such that optimal control gains are computed for each windowed interval independent of the adjacent windows, the transition conditions between two windows are not considered for solving the modified optimal control problem. Hence, the gains for each window have been computed using Ricatti equation without the necessity of accounting for the transition conditions between the solutions for two adjacent windows.

5. Updating weighting matrices by wavelet analysis

It is proposed to update the weighting matrices adaptively by tacking the response states and performing a wavelet-based time-frequency analysis. Let the weighting matrix for the response states be assumed to be identity

$$[Q] = [I] \tag{10}$$

for every time window considered. The weighting matrices for the control effort is updated for every time window by a scalar multiplier and may be written as

$$[R]_i = \delta_i[I] \tag{11}$$

where δ_i is a scalar parameter used to scale the weighting matrix and is obtained based on the time-frequency analysis of a response state. Let $x_k(t)$ be the response state based on which the scaling parameter would be determined. Using DWT with MRA-based application, the response state can be decomposed into time signals resolved at different frequency bands and reconstructed exactly as [6,5]

$$x_k(t) = x_k^{C_L}(t) + \sum_{j=1}^{L} x_k^j(t) \tag{12}$$

where the superscripts denote the different frequency bands, with C_L as the lowest or the coarsest band. The component corresponding to the superscript C_L as $L \to \infty$ represents the non-zero mean component. Any DWT algorithm with a reasonably good localization in time-frequency may be used for this analysis. Based on the MRA the local energy content ratios at different frequency bands over the considered time window (t_{i-1}, t_i) are given by

$$E_{C_L}(t_i) = \int_{t_{i-1}}^{t_i} \{x_k^{C_L}\}^{\mathrm{T}}\{x_k^{C_L}\}\mathrm{d}t \Bigg/ \int_{t_{i-1}}^{t_i} \{x_k\}^{\mathrm{T}}\{x_k\}\mathrm{d}t \tag{13}$$

and

$$E_j(t_i) = \int_{t_{i-1}}^{t_i} \{x_k^j\}^{\mathrm{T}}\{x_k^j\}\mathrm{d}t \Bigg/ \int_{t_{i-1}}^{t_i} \{x_k\}^{\mathrm{T}}\{x_k\}\mathrm{d}t; \quad \forall j = 1, 2, \ldots, L. \tag{14}$$

If the local energy content over a window is above a certain percentage (say p) of the total response energy and/or the band contains one of the natural resonant frequencies of the system, then the weighting on the control effort may be reduced to allow higher control action with less penalty. Hence, if the scalars are varied in the range $(\delta_{\min}, \delta_{\max})$, then, the following conditions may be used to get the values of the weighting scalars over the window

$$\delta_i = \alpha \quad \text{if } E_{C_L}; E_{d_j} \geq p \tag{15a}$$

or

$$\delta_i = \beta \quad \text{if } \omega_{n_i} \leq \omega_{C_L}; \omega_{j-1} \leq \omega_{n_i} \leq \omega_j \tag{15b}$$

or

$$\delta_i = \delta_{\max} \quad \text{otherwise} \tag{15c}$$

where, α and β are suitably chosen scalars in the range ($\delta_{\min}$, $\delta_{\max}$). In Eq. (15), ω_{n_i} is a resonant natural frequency of the system and ω_{C_L} and ω_j are the central frequencies of the different bands.

Once the updated $[R]_i$ matrix is obtained it may be used in the performance functional in Eq. (8) for minimization under the constraint given by Eq. (1) with the control law in Eq. (9) to get the gain matrix. The weighting matrices for the control effort are reduced if the effect of the excitations tracked through the response is found to be significant. This leads to the allowance for additional control effort without penalty. The additional control effort is partly used to nullify the effect of external excitations indirectly. Thus, if the excitation term in Eq. (1) is ignored based on the assumption that the weighting matrices are indirectly based on tracking the response time-frequency characteristics over the window, then for each window the minimization of the performance functional leads to the Ricatti algebraic equation under steady-state condition. From this, the gain matrix $[G]_i$ could be solved and updated for each window with updated weighting matrix $[R]_i$.

6. Numerical results

SDOF system

To illustrate the potential application of the proposed time-varying LQR updating scheme a few different representative earthquake ground motions have been considered for numerical simulations. These earthquake ground motions typically represent near fault motions for which controlling the response of the structures is known to be more challenging due to their sudden impulsive type of action. The data considered are recorded accelerograms for (i) the N–S component of the 1979 Imperial Valley earthquake at El Centro site (ii) the fault normal component of the 1994 Northridge earthquake at Newhall site (iii) the E–W component of the 1995 Kobe earthquake at the KJMA site. These three accelerograms are used as base excitations to simulate the response of a SDOF system. The SDOF system considered is assumed to have a natural period of 1 s and a modal damping ratio of 1%. Both the conventional LQR and the proposed time-varying adaptive LQR formulation are used separately on the SDOF system to control the response and observe the effect of the proposed time-varying adaptive control scheme in comparison with the conventional LQR controller.

For the examples considered here, the parameter α (Eq. (15a)) is not used. The two parameters used for scaling the weighting matrices are β and $\delta_{\max}$. These parameters are also predefined. Hence, for application of the proposed control scheme the possible set of gain matrices can be computed offline based on the values of these parameters used for scaling the weighting matrices. Thus, for implementation of the scheme

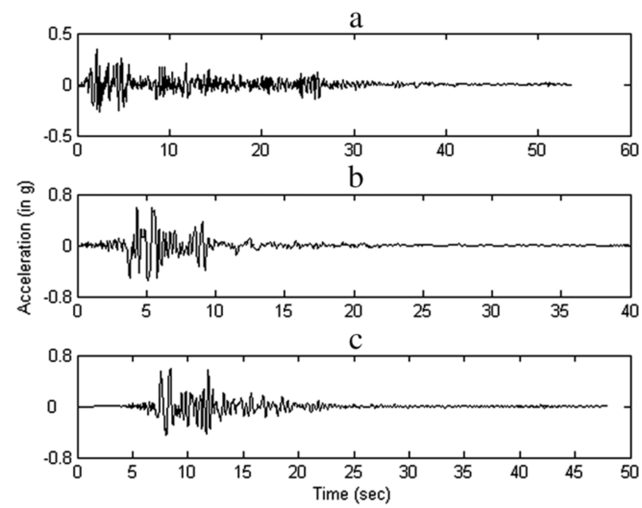

Fig. 1. Ground acceleration time histories for (a) 1979 Imperial Valley earthquake recorded at El Centro site, N–S component (b) 1994 Northridge earthquake recorded at Newhall site, fault normal component (c) 1995 Kobe earthquake recorded at KJMA site, E–W component.

the only online calculation required is the central frequency of the dominant frequency band of the response over the windowed interval. Consequently, the appropriate gains can be applied from the set of gains calculated a priori.

Fig. 1 shows the plot of the three accelerograms. All the time histories are characterized by the arrival of a low frequency pulse at the beginning of the records followed by high frequency rich accelerations. The controlled response of the SDOF system under each of these excitations is computed first of all. Subsequently, the controlled responses using (a) the LQR and (b) the proposed adaptive LQR algorithms are evaluated. For the LQR scheme, the weighting matrix $[Q]$ has been chosen to be identity and the value of R has been chosen as 1. In the case of the adaptive LQR algorithm, the weighting scalar $\delta_i = \beta\ (=R)$ is chosen as 0.1 if the maximum response energy is contained in either of the two bands for which the central frequencies are closest to (above and below) the natural frequency of the SDOF system. Otherwise, the value of $\delta_i\ (=R)$ is kept as 1 ($=\delta_{\max}$). The matrix $[Q]$ is chosen as identity. The wavelet analysis of the windowed response is carried out using the Daubechies' D4 DWT, which has relatively good localization in time and frequency with sufficient smoothness. The interval window is considered as 1 s for updating. Based on the wavelet analysis of the response over a time interval of 1 s, the weighting parameters are decided following the previously mentioned scheme. The Ricatti equation is solved for each of these windows to update the gain matrices and calculating the associated control forces and the controlled responses. All computations have been carried out on MATLAB and it has been observed that the computational delay associated with the proposed control scheme is insignificant (of the order of 0.01 s) as compared to the interval time of updating.

Fig. 2 shows all the relevant results for the case of 1979 Imperial Valley earthquake excitations. Fig. 2(a) shows the uncontrolled displacement response and Fig. 2(b) shows the controlled response corresponding to the conventional LQR and

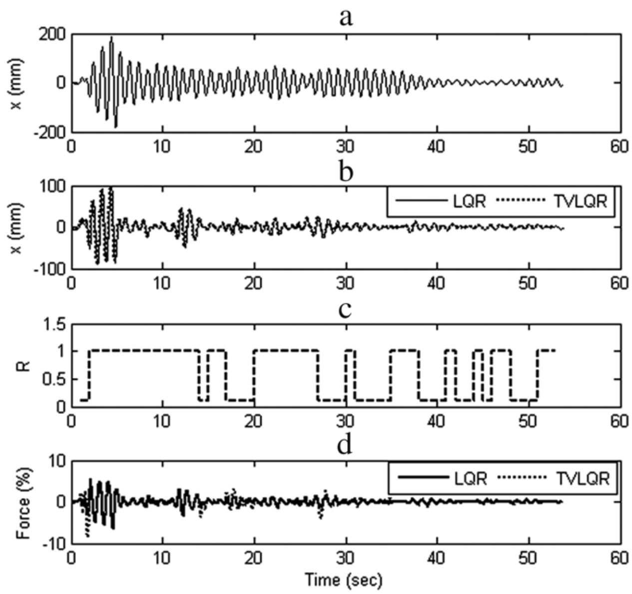

Fig. 2. Results for the 1979 Imperial Valley case: (a) uncontrolled displacement response (b) controlled response using LQR and time-varying LQR algorithms (c) variation of R with time (d) normalized control forces with respect to the total weight for the conventional LQR and the proposed time-varying LQR control.

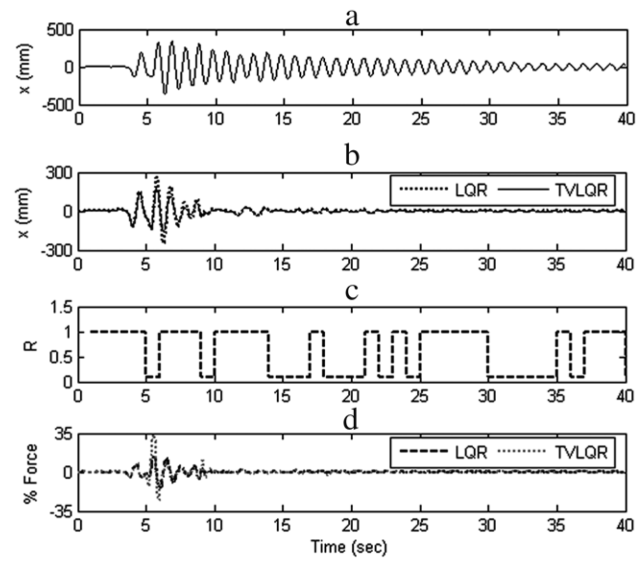

Fig. 3. Results for the 1994 Northridge case: (a) uncontrolled displacement response (b) controlled response using LQR and time-varying LQR algorithms (c) variation of R with time (d) normalized control forces with respect to the total weight for the conventional LQR and the proposed time-varying LQR control.

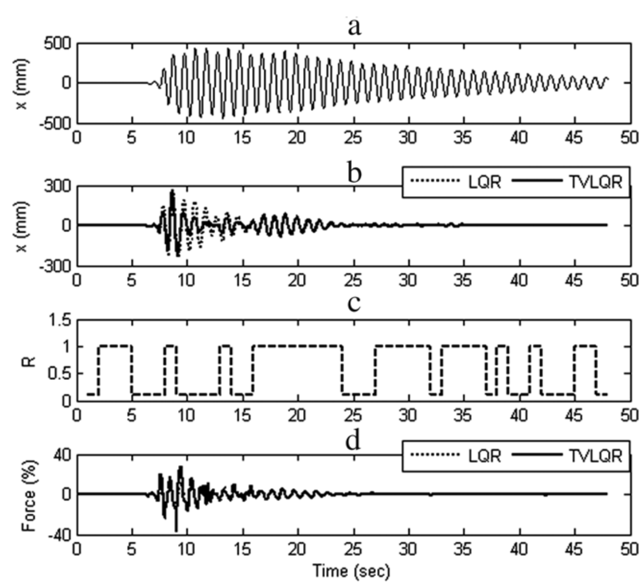

Fig. 4. Results for the 1995 Kobe (E–W) case: (a) uncontrolled displacement response (b) controlled response using LQR and time-varying LQR algorithms (c) variation of R with time (d) normalized control forces with respect to the total weight for the conventional LQR and the proposed time-varying LQR control.

the wavelet-based time-varying adaptive LQR control schemes. The variation in the value of R is plotted in Fig. 2(c). The controlled response obtained using the two control schemes are comparable in this particular case and so are the control forces (normalized to the weight of the structure) as seen from Fig. 2(d). The maximum displacement is reduced to 100 mm from the maximum uncontrolled displacement of about 200 mm by the application of a peak control force of about 8.5% of the total weight. It is not unexpected that the two control schemes perform comparably in this case, this being evident from the plot of R with time in Fig. 2(c). For most of the time, during the strong motion phase between 0 and 15 s the value of $R = 1$, which coincides with the case of classical LQR problem.

To examine the effectiveness of the proposed wavelet-based time-varying LQR control more critically, the ground motions corresponding to the 1994 Northridge earthquake is considered next. The results similar to those in Fig. 2 are plotted in Fig. 3. The effectiveness of the proposed wavelet-based adaptive LQR scheme as compared to the conventional LQR control is evident in this example with the peak displacement reduced to about 180 mm for the time-varying LQR case as opposed to the peak controlled displacement of 270 mm achievable by LQR control. The peak control force requirement for the time-varying LQR case is about 34% while that for the LQR is about 15%. Though, the peak control force requirement is larger for the proposed time-varying LQR (TVLQR) control, the control forces for the two algorithms are comparable for most of the times over the duration and the TVLQR control forces are even smaller at times. This indicates that the total energy demands for the two control schemes are comparable even though the peak force for the proposed adaptive wavelet-based TVLQR controller is higher.

Similar plots as in the previous figures are shown in Fig. 4 for the case corresponding to the excitations for the E–W component of the 1995 Kobe earthquakes at the KJMA station. With the E–W component of the KJMA accelerations, the controlled displacement response with the TVLQR controller is reduced significantly after the peak displacement response occurs as compared to the displacement reduction achieved by the conventional LQR controller, though with an increased peak control force. In this case too, the total energy demand

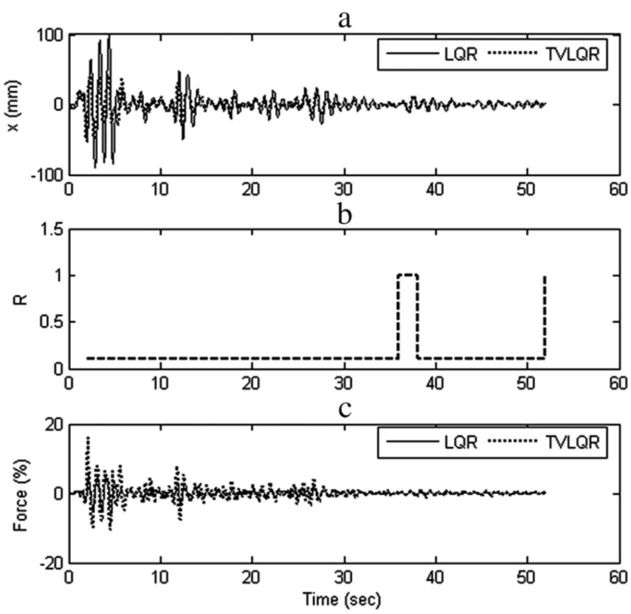

Fig. 5. Modified updating interval (2 s) for the case of 1979 Imperial Valley earthquake (a) controlled response using LQR and time-varying LQR algorithms (b) variation of R with time (c) normalized control forces with respect to the total weight for the conventional LQR and the proposed time-varying LQR control.

for the two controllers are not very different apart from the instantaneous larger peak control force demand for the proposed TVLQR controller (18% for the LQR and 36% for the TVLQR).

Updating interval

In all the previous examples considered the updating interval has been assumed to be 1 s. The updating interval has been decided upon based on the practical consideration in implementation of the updating as well as (i) expected range of local frequency content which is dependent on the excitations (in this case earthquakes) and (ii) the possible range of the structural natural period.

To study the effect of the updating interval on the controlled response and on the magnitude of the control force requirement an updating interval of 2 s is chosen. It has been observed in all the cases that with marginal to moderate increase in the peak control force requirement, the maximum controlled displacements have been significantly reduced. Figs. 5 and 6 show the results for the cases of El Centro and the E–W component of the KJMA ground motions respectively. On comparing Figs. 2 and 5 it is observed that the peak controlled displacement has been reduced approximately by half with a marginal increase of peak control force by about 7%. The results in Fig. 6 are more striking. The peak displacement (as compared from Fig. 4) is reduced by about 40% with almost no increase in the peak control force.

3 DOF system

A 3 DOF system is considered as the next example to illustrate the application of the proposed control strategy in the case of MDOF systems. This represents a typical civil engineering structural system and also allows the installation of multiple controllers to investigate their effect.

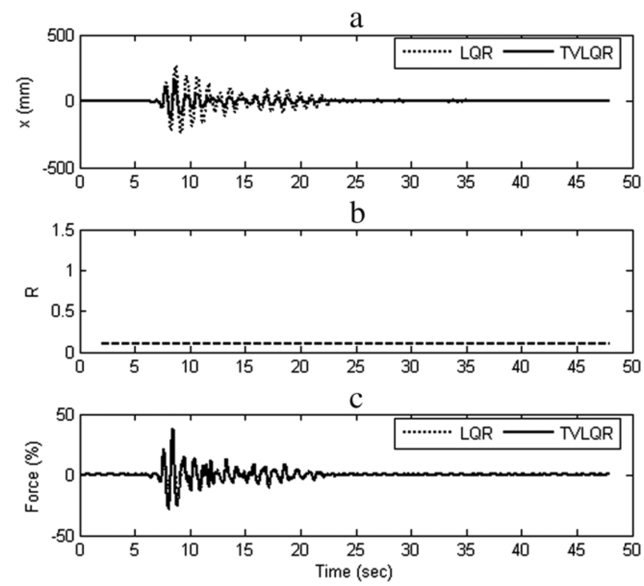

Fig. 6. Modified updating interval (2 s) for the case of 1995 Kobe earthquake (E–W component) (a) controlled response using LQR and time-varying LQR algorithms (b) variation of R with time (c) normalized control forces with respect to the total weight for the conventional LQR and the proposed time-varying LQR control.

A typical shear building with three stories having the masses lumped at each floor is considered. The masses at each floor are assumed to be 10 000 kg and the stiffnesses at each floor are uniformly assumed to be 2000 kN/m. The natural frequencies of this 3 DOF system are 1.0 rad/s, 2.81 rad/s and 4.06 rad/s. The damping for each mode is uniformly assumed to be 1%. The uncontrolled displacements for the three floors are plotted in Fig. 7 when subjected to the Newhall ground accelerations. A single controller is applied at the top floor for the first case. The controlled responses using the LQR and the proposed TVLQR control algorithms are computed and plotted in Fig. 8. For the TVLQR control scheme updating interval is considered as 2 s based on the previous conclusions for the SDOF system. The TVLQR-based control algorithm significantly reduces the response at all three floor levels (Fig. 8(a)–(c)) as compared to the responses simulated using the LQR controller. The control force time histories (in Fig. 8(d)) compare closely for the two controllers, though the instantaneous value of the peak control force for the TVLQR controller is around 25% (compared to 11% in the case of LQR controller). It may be possible to reduce the peak control force values by introduction of multiple controllers, which is examined in the next case.

The response of the 3 DOF system considered in the last example is evaluated again with three controllers, one placed at each floor level. The controlled responses at each floor level, using LQR and TVLQR control algorithms, when the system is subjected to the 1994 Newhall ground accelerations are plotted in Fig. 9. The corresponding control force time histories for each of the controllers normalized as a percentage of the floor weight are shown in Fig. 10. The controlled displacement responses compare with the case of a single controller (Fig. 8(a)–(c)), but it is observed that the use of three controllers reduces the peak control force requirement

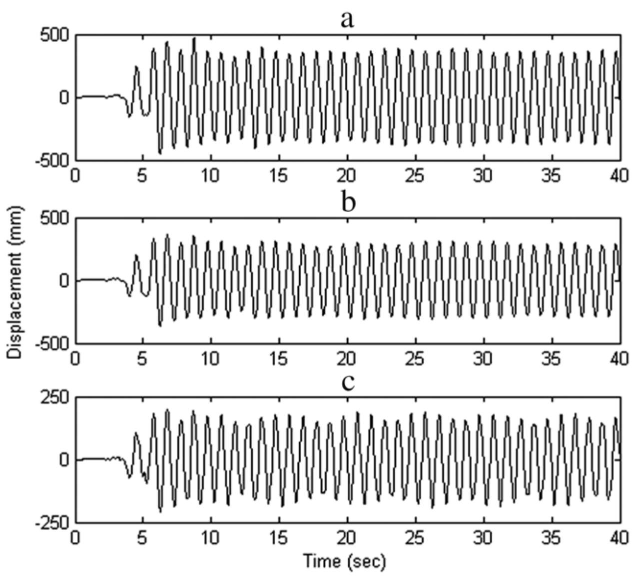

Fig. 7. Uncontrolled displacement responses for the 3 DOF system subjected to Newhall ground motions (a) third floor (b) second floor (c) first floor.

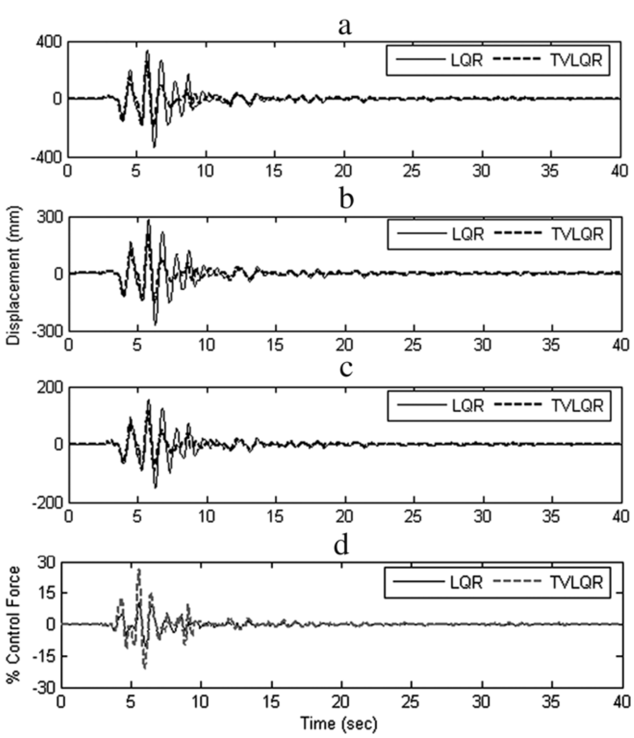

Fig. 8. Controlled response of the 3 DOF system with one controller at top floor using LQR and time-varying LQR schemes (a) at third floor (b) at second floor (c) at first floor (d) Control forces as a percentage of the total weight for the LQR and the time-varying LQR algorithms.

both for the LQR and the proposed wavelet-based adaptive TVLQR control for each of the controllers. The peak control forces for the controllers at the third (top), second and first floors for the proposed TVLQR controller are about 16% (7% for LQR controller), 11% (6% for the LQR controller) and 6.5% (2.5% for the LQR controller) respectively. This shows that the peak control force demands for the case with multiple

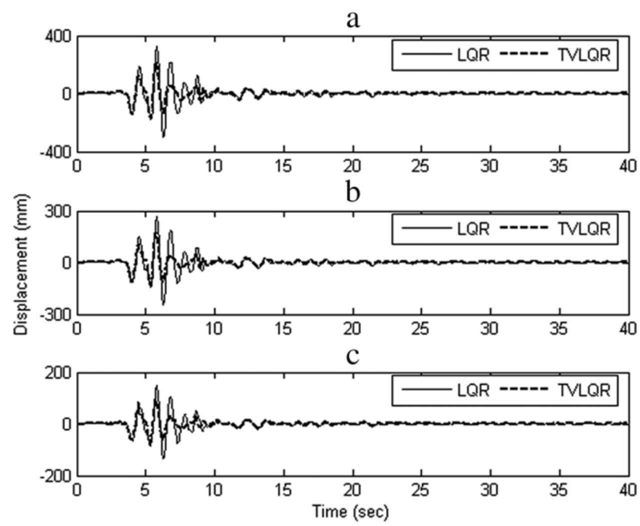

Fig. 9. Controlled response of the 3 DOF system using LQR and time-varying LQR schemes (a) at third floor (b) at second floor (c) at first floor; with three controllers, one at each floor.

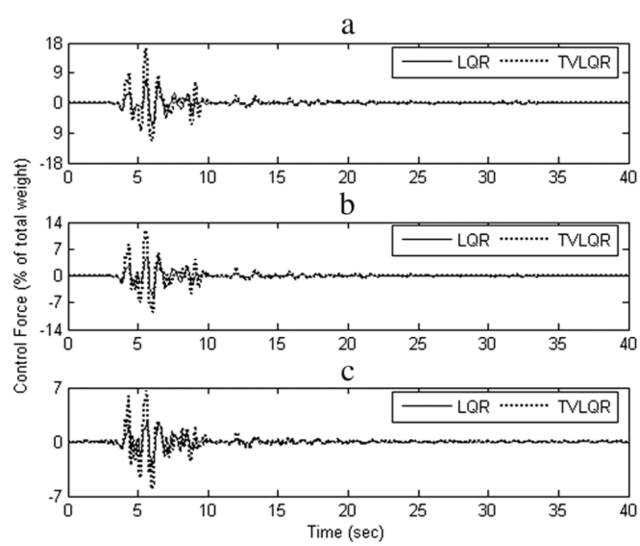

Fig. 10. Control forces as a percentage of floor weight for the case of the conventional LQR and the proposed time-varying LQR algorithms, for the controller at (a) third floor (b) second floor (c) first floor.

controllers have been brought down to reasonable limits. Additionally, the differences in the peak control forces between the proposed TVLQR and the conventional LQR controllers are not significant anymore to outweigh the advantage of significant additional displacement reduction achievable with the wavelet-based adaptive TVLQR controller.

The proposed TVLQR controller has the inherent capability and flexibility to account for the variability in the nature of the response by deriving information using wavelet analysis and incorporating the information through an adaptive control scheme. The performance of the proposed adaptive time-varying controller is better in terms of reducing the displacement response of structures in some cases as compared to the classical LQR controller and in the worst case it achieves similar reduction as in the case of an LQR controller.

7. Conclusions

A wavelet-based adaptive time-varying controller has been proposed and investigated in this paper. The controller is designed by modifying the conventional LQR controller by updating the weighting matrices applied to the response energy and the control effort to solve the optimal control problem, over time intervals. The weighting matrices are decided adaptively through the wavelet analysis of the response locally in time to account for the time-varying frequency distribution of the energy content representing the effect of the non-stationary ground excitations on the structural dynamic system. This eliminates the apriori requirement of deciding on the weights usually carried out arbitrarily in the classical LQR case. The proposed modified LQR controller with adaptive gains has been used to simulate the controlled responses of SDOF and MDOF systems with single and multiple controllers.

Several representative recorded earthquake accelerograms have been chosen as excitations for simulation of the responses. The chosen ground motions are typical near fault motions that are used to investigate the effectiveness of the proposed algorithm. The proposed wavelet-based adaptive LQR controller performs significantly better than the conventional LQR controller in a number of cases in reducing the displacement response of the structure and achieving similar displacement reduction as with the LQR controller in the worst case. However, for SDOF systems the peak control force for the proposed wavelet-based adaptive TVLQR is greater than that for the conventional LQR controller (instantaneously), though the total control energy demand for both are comparably. The effect of the updating time interval has been investigated and the dependence of the control efficiency on this parameter has been revealed. For the case of MDOF systems, the proposed wavelet-based adaptive TVLQR controller has shown its effectiveness in controlling the displacement responses as compared to the LQR controller, both in the case of single and multiple controllers. In particular, the case with multiple controllers achieves significant response reduction (as compared to those obtained by the LQR controller) with acceptable levels of control forces, which are not much greater than the control force requirements for the LQR controller. The inherent flexibility in the design of the proposed adaptive controller to account for the variation in the frequency content of the local energy in time via the modification of the LQR controller makes it an attractive controller for seismic vibration control of structures.

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